SUSTAINABLE WATER MANAGEMENT IN THE MINERALS INDUSTRY

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Abstract

The problem of managing a storage dam subject to an irregular input and with the possibility of using an alternative source is of considerable interest. It arises in the provision of water for Queensland coal mines, where additional water is available via a pipeline from a public supply, and also in cases where recycled or more expensive water is used to supplement the normal supply. We investigated discrete and continuous probability formulations, simulation methods, and the development of possible control policies. It was determined that without some feedback control of the net flows, the dam will eventually empty or overflow. A policy that uses the additional supply to maintain a low probability of the dam going empty at future times is recommended.

1. Introduction

In the operation of a coal mine, water is an important resource, without which the mine cannot operate. Central Queensland coal mines collect rain water in dams and also have access to a water pipeline. The supply of rainwater varies with the season and from year to year. There is also a considerable amount of evaporation from the storage dams. The mines use both fresh water and recycled (used) water. The major uses of the water are in eliminating impurities from the coal (coal washing), in dust suppression particularly on the roads, and in underground workings. Recycled water is used for coal washing and dust suppression, while fresh water is needed for the underground workings. The coal washing

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plant separates waste rock from the coal, and returns a significant proportion of its feed water to the used water dam. This return (used) water has an increased salt content from the washing process.

There are two major problems related to maintaining a continuous supply of water to the mine. These are what size of dam is needed, and how should the use of pipeline water be scheduled so as to best avoid running out of water, and overflow of water. An additional problem is the amount and control of salt content in the used water dams.

While there may be several dams at a given mine this report considers them as a total volume of stored water, or in the case of the simulation of salt build up the dams are divided into a fresh-water dam and a used-water dam.

Data was available on rainfall and evaporation on a monthly basis for the past 40 years in the coal mining region of interest.

2. Literature

The problem of maintaining a dam level is similar to inventory problems and these have been studied extensively [1, 2]. There are also studies of dam problems [2, 3, 4, 5, 6, 7, 8]. In both cases assumptions about the variability of the inputs and or outputs assume a particular distribution. In the case being studied at the MISG the variability of the rainfall does not follow the simple distributions assumed in the literature.

Also, the literature on dam levels concentrates on the probability of the dam becoming empty, where the mine dam problem also needs to consider the case of the dam overflowing as the dam contents are often not of sufficient purity for discharge into the environment.

An interesting result given by Kendall [4] is a calculation of the probability of an infinite dam eventually emptying, when the average input I is only slightly greater than the usage U, the initial dam content is S, and the standard deviation of the input is σ. This probability is:

$$\exp\left(-\frac{2S}{U}\left(\frac{I}{U}-1\right)\left(\frac{I}{\sigma}\right)^2\right) \tag{1}$$

It is seen that this probability decreases when the initial content S increases relative to U, the input I increases relative to U, and σ decreases relative to the input flow I.

3. Rainfall and evaporation

The Central Queensland coal mines are in an area of variable rainfall and high evaporation. Data on rainfall and evaporation from 1961 to 2002 was available, and a monthly estimate of the proportion of the rainfall that would run to the dams. Rain in this region falls mainly in the summer months (December to March) and is quite variable with a standard deviation similar to the mean. Evaporation is more consistent with a standard deviation about one tenth of the mean. Figure 1 gives the rainfall and Figure 2 shows the evaporation (both rainfall and evaporation are conveniently expressed for this work as mm per month).

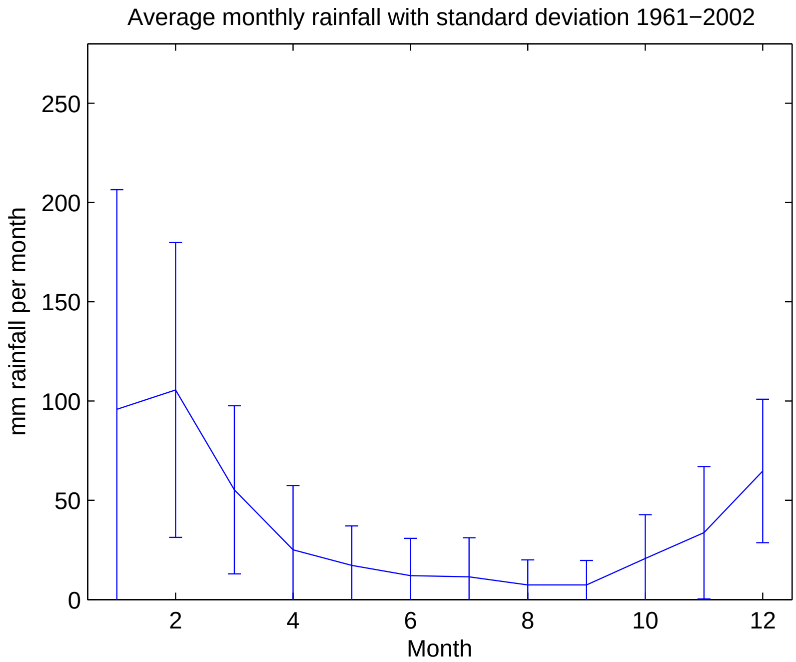

Figure 1. Annual rainfall for typical Central Queensland coal mine.

Evaporation is higher than rainfall over most months. It determines how much water runs into the dam and losses from the area of the dam. During the dry months the catchment area is dry and there is very little runoff into the dam. Monthly values for the fraction of rainfall in the catchment area that can be expected to runoff into the dam were supplied as given in Table 3.1

Jan	Feb	Mar	Apr	May	Jun
0.195	0.245	0.136	0.044	0.03	0.023
Jul	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
0.02	0.006	0.012	0.057	0.10	0.132

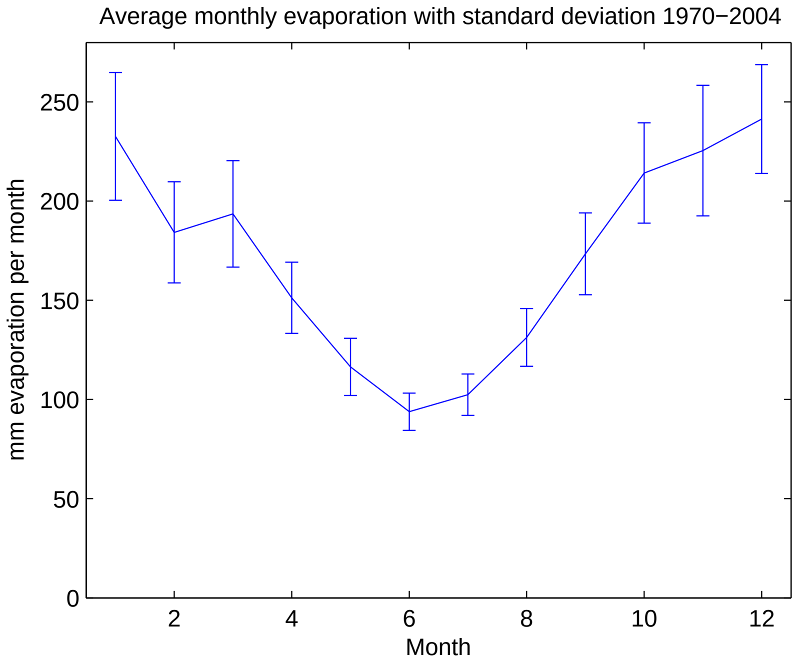

Figure 2. Annual evaporation for typical Central Queensland coal mine.

Table 3.1: Fraction of rainfall reaching dam by the month

Table 3.1 shows a considerable variation in runoff fraction over the year but are assumed constant for a given month. However, the factors may be too simplistic as the amount of rain during a given month varies by a large amount. A simple model of the amount of water in the catchment area to get more reliable estimates of runoff into the dam may be appropriate.

During the wet season months, the rainfall follows an approximately log normal distribution (Figure 3), while the evaporation can be considered to follow a normal distribution truncated at the high end corresponding to a maximum evaporation (Figure 4). These approximations could be improved if found necessary in a more detailed examination of the data. However, they are sufficient to demonstrate how the rainfall data can be used in capacity calculations.

The lower rainfall and the very low runoff fraction figures given above indicate that in the dry season April to November there is essentially no runoff into the dam.

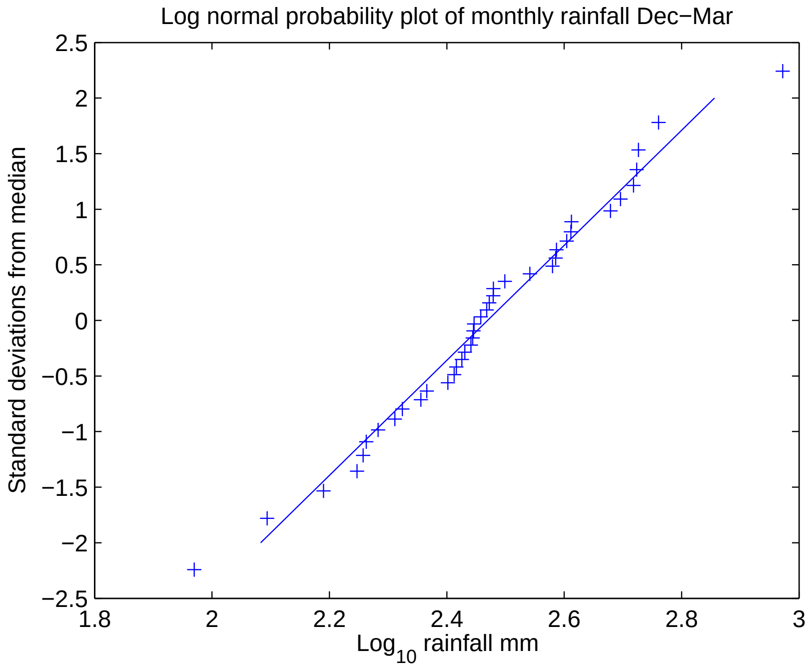

Figure 3. Wet season rainfall on a log normal probability plot.

4. Order of magnitude estimates

To obtain a feeling for the scale of the water storage and usage at a typical mine some order of magnitude estimates were attempted and are shown in Table 4.1.

Consumption:	$\sim 5 \times 10^5$ m^3/month
Rainfall:	$<= 2 \times$ consumption ($\sigma = \mu$)
Evaporation:	$\sim 0.2 \times$ consumption ($\sigma = 0.1\mu$)
Return to worked store:	$\sim 0.1 \times$ consumption
Storage capacity:	~ 2 years
Pipeline supply:	$\sim 0.25 \times$ consumption
Seepage:	$\sim 0.1 \times$ consumption

Table 4.1: Order of magnitude estimates for storage flows.

As it proved difficult during the MISG to obtain consistent estimates of areas of dams and the catchment areas, details of these were not used, and instead the work concentrated on the generic effects of the dam inputs and outputs.

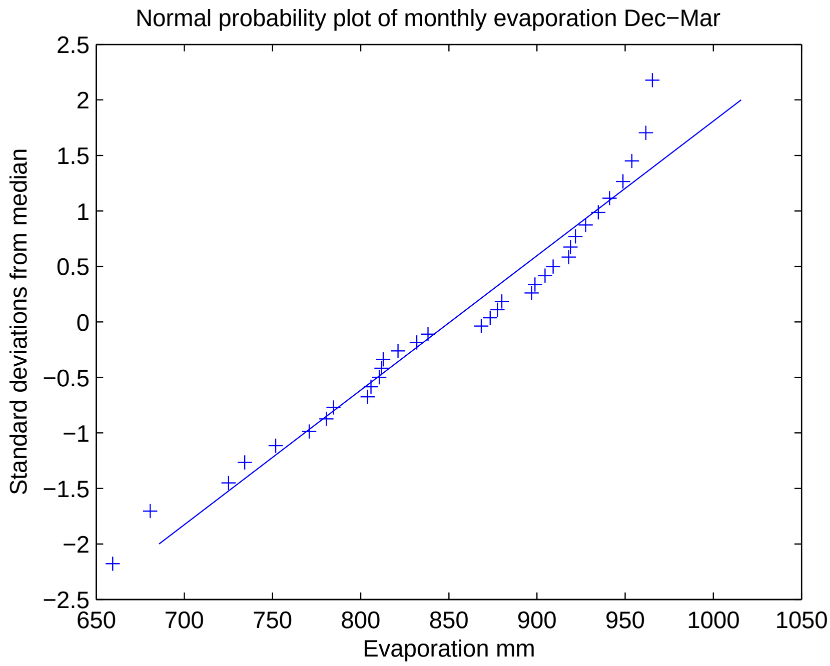

Figure 4. Wet season evaporation on a normal probability plot.

The variation in the water balance at the dam is totally dominated by the variation in rainfall, and thus evaporation and water usage can be considered essentially constant.

5. Random walk and the need for feedback

The net input to the dam can be considered a random variable that is being integrated to give the dam level. This results in a random walk, possibly on top of a net trend due to the net input not being zero, that gives the dam level. Even without a trend in the dam level, the water level will always eventually reach either zero or its maximum as the variance of a random walk increases with time until a limit is reached.

To ensure that the dam neither empties or overflows some control of the inputs (or outputs) is required. As there is a major random variation in the dam input the control can only reduce the probability of reaching a limit to an acceptable level. Two possible controls exist. The first is the amount of water taken from the pipeline to reduce the probability of the dam running empty. The second prevents overflow by bypassing rainfall around the dam, or dumping water from the dam.

Where the net input without the pipeline is negative, increasing the dam size reduces the probability of a dam overflow. Similarly a larger dam, provided net long term input is sufficient, reduces the probability of the dam becoming empty.

In times of excessively low dam level, measures to conserve water may be implemented providing another means of control.

6. Probability based formulations

In this section both discrete and continuous formulations based on probabilities are considered. It will be seen that both cases end up with very similar expressions.

6.1. Discretised dam levels

The level in the dam can be formulated as a Markov chain as follows: Assume that the dam level is divided into n discrete levels of width δx so that the i^{th} level is from height $x = (i-1)\delta x$ to $x = i\delta x$. Then $p_{i,t}$ is the probability of being in level i at time t. The probability of moving from level $i-1$ to level i can be expressed in two parts:

The probability of moving between level $i-1$ and level i due to the mean flow is: $\delta t\, f_{i-1/2}\,(p_{i-1,t}+p_{i,t})/(2\,\delta x)$ which is in the direction of f the rate of rise of the dam level. The divisor δx is introduced to make the factor f independent of the size of the discrete levels.

The probability of moving into and out of the i^{th} section due to random changes in level is: $\delta t\, s_{i-1}\, p_{i-1}/(\delta x)^2$ in the positive direction and $\delta t\, s_i\, p_i/(\delta x)^2$ in the reverse direction. s determines the amount of random variation in the dam level. In this case a divisor of δx^2 is needed to make s independent of the size of the the discrete levels.

Hence for the change in probability for small time step δt:

$$\begin{aligned} p_{i,t+\delta t} \;=\; & p_{i,t} + \delta t\,\{f_{i-1/2}\,\frac{p_{i-1,t}+p_{i,t}}{2}/(\delta x) \\ & -f_{i+1/2}\,\frac{p_{i,t}+p_{i+1,t}}{2}/(\delta x) \\ & +s_{i-1}\,p_{i-1,t}/(\delta x)^2 - s_i\,p_{i,t}/(\delta x)^2 \\ & -s_i\,p_{i,t}/(\delta x)^2 + s_{i+1}\,p_{i+1,t}/(\delta x)^2\} \end{aligned} \qquad (2)$$

At the boundaries there is only the possibility of staying in the boundary segment or moving away from the boundary giving the equations for the change in probability at the boundaries:

$$p_{1,t+\delta t} = p_{1,t} + \delta t(-f_{1+1/2}\,\frac{p_{1,t}+p_{2,t}}{2}/(\delta x) - s_1\,p_{1,t}/(\delta x)^2 + s_2\,p_{2,t}/(\delta x)^2) \tag{3}$$

$$\begin{aligned} p_{n,t+\delta t} &= p_{n,t} + \delta t(f_{n-1/2}\,\frac{p_{n-1,t}+p_{n,t}}{2}/(\delta x) + s_{n-1}\,p_{n-1,t}/(\delta x)^2 \\ &\quad -s_n\,p_{n,t}/(\delta x)^2) \end{aligned} \tag{4}$$

These equations (1 - 3) can be written in matrix form:

$$\mathbf{p}_{t+\delta t} = \mathbf{p}_t + \delta t\,\mathbf{A}\mathbf{p}_t \tag{5}$$

Where in the case of constant coefficients:

$$\mathbf{A} = \begin{bmatrix} -S-F & S-F & 0 & 0 & .. & 0 & 0 & 0 \\ S+F & -2S & S-F & 0 & .. & 0 & 0 & 0 \\ 0 & S+F & -2S & S-F & .. & 0 & 0 & 0 \\ . & .. & .. & .. & .. & .. & .. & \\ 0 & 0 & 0 & 0 & .. & S+F & -2S & S-F \\ 0 & 0 & 0 & 0 & .. & 0 & S+F & -S+F \end{bmatrix}$$

where

$$S = s_{i,t} \text{ and } F = f_{i+1/2,t}/2$$

It is easily verified that for steady state this has the solution:

$$p_i = K\left(\frac{S+F}{S-F}\right)^i \tag{6}$$

where K is chosen so that $\Sigma p_i = 1$. For the case where f_i or s_i is not constant with respect to i, the steady state version of equation 5 is still easily solved for the probabilities p_i as the required equation is homogeneous and tridiagonal.

The equations 1 - 3 can also be rewritten as:

$$\begin{aligned} (p_{i,t+\delta t} - p_{i,t})/\delta t &= (f_{i-1/2}\frac{p_{i-1}+p_i}{2} - f_{i+1/2}\frac{p_i+p_{i+1}}{2})/(\delta x) \\ &\quad +\{(s_{i-1}p_{i-1,t} - s_i p_{i,t})/(\delta x) \\ &\quad -(s_i p_{i,t} - s_{i+1}p_{i+1,t})/(\delta x)\}/(\delta x) \end{aligned} \tag{7}$$

$$\delta x(p_{1,t+\delta t} - p_{1,t})\delta t = -f_{1+1/2}\frac{p_1+p_2}{2} - (s_1 p_{1,t} - s_2 p_{2,t})/(\delta x) \tag{8}$$

$$\delta x(p_{n,t+\delta t} - p_{n,t})/\delta t = f_{n-1/2}\frac{p_{n-1}+p_n}{2} + (s_{n-1}p_{n-1,t} - s_n p_{n,t})/(\delta x) \tag{9}$$

Taking the limit as both δx and δt go to zero and putting $x = i\,\delta x$ and $x_n = n\,\delta x$ (so $p_{i,t} = p(x,t)$) gives:

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(-f(x)\,p(x,t) + \frac{\partial s(x)p(x,t)}{\partial x}\right) \tag{10}$$

$$0 = -f(0)\,p(0,t) + \frac{\partial s(0)p(0,t)}{\partial x} \tag{11}$$

$$0 = f(x_n)\,p(x_n,t) - \frac{\partial s(x_n)p(x_n,t)}{\partial x} \tag{12}$$

For the steady state $\partial p(x,t)/\partial t = 0$ and constant coefficients f and s the equation:

$$p(x) = K\exp(xf/s) \tag{13}$$

satisfies both the main equation (10) and the boundary conditions (11) and (12). The value of K is such that this expression becomes a probability distribution i.e.:

$$\begin{aligned} K &= 1/(\int_0^{x_n} \exp(xf/s)dx) \\ &= f/\{s(\exp(x_n f/s) - 1)\} \end{aligned} \tag{14}$$

The conditions of empty and of overflowing, need for this formulation to be defined as, say, the bottom and top 5% of the range, as the probabilities of being exactly empty and full are given as zero. This formulation has not included an adequate formulation of the behaviour at the empty and full conditions, and has not allowed for the asymmetry in the rainfall distribution.

6.2. Wiener processes and Fokker-Planck equation

The stochastic differential form for a Wiener process [9] is:

$$dV = Fdt + \sigma dW \tag{15}$$

where V is the volume in the dam, F is the rate of volume change due to flow into the dam, σ is the standard deviation of F, and dW is the stochastic derivative term. Introducing the probability of being at level V at time t, $p(V,t)$ this leads to the forward Fokker Planck equation:

$$\frac{\partial p(V,t)}{\partial t} = \frac{\partial}{\partial V}\left\{-Fp(V,t) + \frac{1}{2}\frac{\partial}{\partial V}\left\{\sigma^2 p(V,t)\right\}\right\} \tag{16}$$

and the boundary conditions:

$$-Fp(0,t) + \frac{1}{2}\frac{\partial}{\partial V}\left\{\sigma^2 p(0,t)\right\} \tag{17}$$

$$-Fp(V_{max},t) + \frac{1}{2}\frac{\partial}{\partial V}\left\{\sigma^2 p(V_{max},t)\right\} \tag{18}$$

Similar to the previous subsection, these have an exponential solution for steady state with constant F and σ:

$$p(V) = K\exp(VF/\sigma^2) \tag{19}$$

and thus have the same problems as noted above. To handle the actual rainfall properties and the boundary conditions a simulation based formulation was investigated.

7. Simulation based formulation

An alternative approach is to run a Monte Carlo simulation of the dam level. Similar to the discrete formulation above, the dam contents are described by a number of discrete levels, and a probability of moving from one level to another is applied. As the simulation is run the time spent in each of the discrete levels is recorded. For level i the probabilities of moving to level $i-1$ and to level $i+1$ are defined as:

For net flow in a small time interval as a probability of p_f of moving to the level above if net flow is positive, and to the level below if net flow is negative.

For random variation in the net flow a probability of p_s of moving to the level above and also to the level below.

At the upper boundary it is assumed any excess water overflows, while at the lower boundary no water can be withdrawn, so that the simulation is limited to feasible dam levels.

The simulation is started by assuming an initial value for the dam level. For each time step, a random number $(0 < r < 1)$ is used to determine if the level changes to the next level due to net flow $(r < p_f)$, and a second random number $(0 < r < 1)$ is used to determine if a drop to the level below occurs $(r < p_s)$, or an increase to the level above occur $(r > 1 - p_s)$. The levels generated by the simulation are recorded to determine the distribution of occurrence of the dam levels.

Another approach to the stochastic simulation of dam levels is to use a continuous level measurement and at each time step add the net change in level and a continuous random variable to account for the random variation in the flow. This, however, creates a non-zero probability of being at the zero level and the maximum level that decreases as the time step decreases.

8. Determination of dam size

As noted above, the rainfall in central Queensland can be divided into a wet season (December to March) and a dry season (April to November). Advantage can be taken of this in creating a simulation that considers only two parts each year.

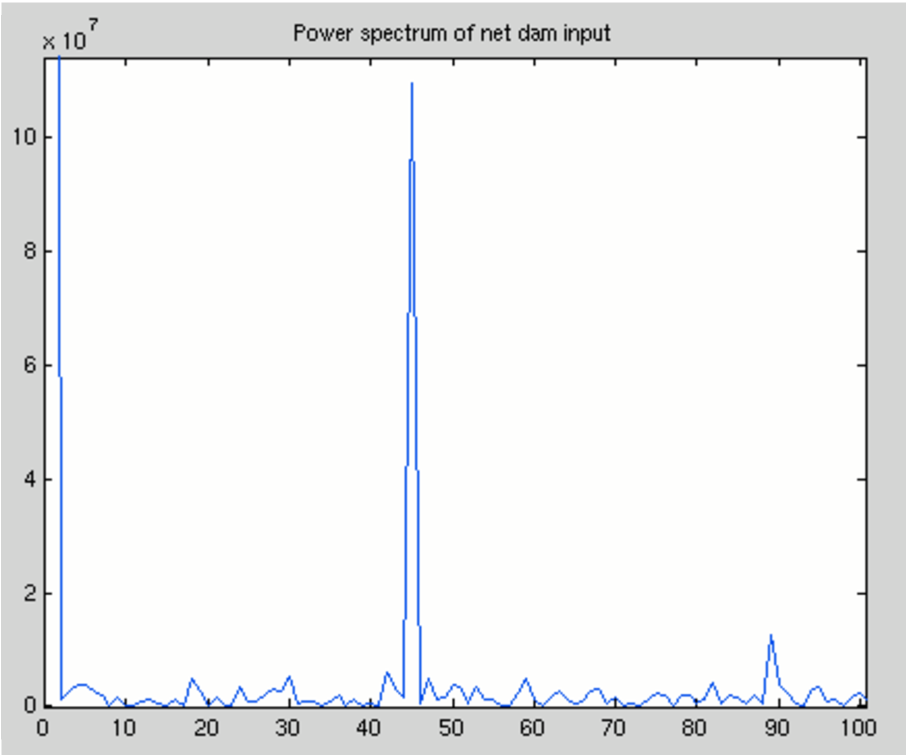

Figure 5. Power spectrum of net dam input. The initial spike is the constant term, and the two other spikes are one cycle per year and the smaller two cycles per year.

An examination of the rainfall during the wet season found it to closely follow a log Normal distribution (Figure 3). The Fourier analysis (Figure 5) indicates very limited correlation between years. So it seems sufficient to generate a sample wet season rainfall independently. The dry season runoff is essentially zero. A balance between supply and consumption in the long term is assumed.

The time the dam is empty (and hence the proportion of time it is empty) is calculated as the sum of the lengths of time during the dry season that the dam is empty, calculated as for each dry season as:

$$T = \begin{cases} -h_1/(h_0 - h_1) & \text{when } h_1 < 0 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

where h_0 is the initial dam level and h_1 is the calculated final level or deficiency in level. A similar formula is used for the time in overflow condition in the wet season.

It is then possible to determine the time over which empty and overflow conditions occur in a simulation run for a dam of a given size, and to produce a plot of the probabilities of empty and full conditions. Figure 6 gives an example of such a plot.

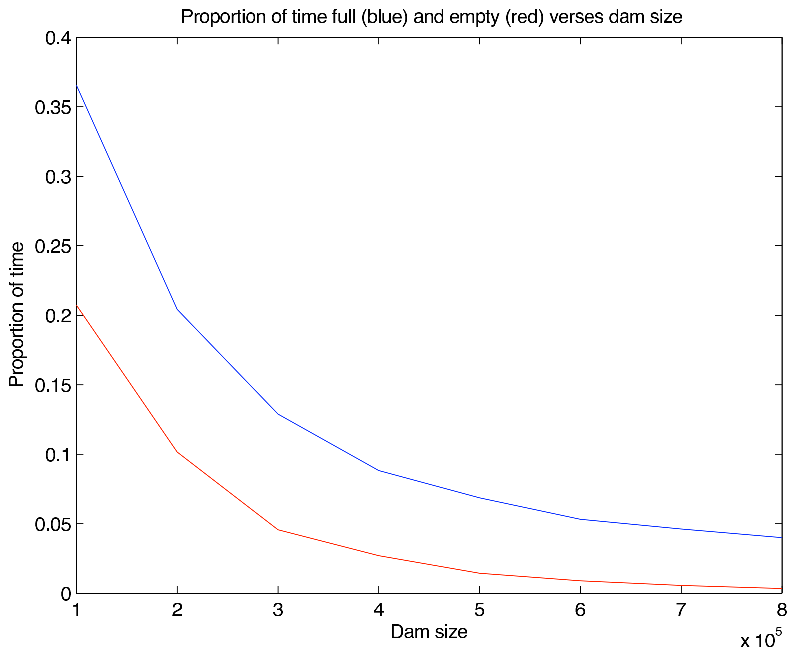

Figure 6. Typical proportions of time for empty and overflow conditions as a function of dam size.

9. Bootstrap testing of control policies

The MISG group investigated several policies for the use of pipeline water. It is not known what criteria should be used to evaluate the different policies. In fact the criteria will certainly vary from one mine to another. The two main terms in the evaluation are the probabilities of running empty and of overflowing.

A Pareto optimum graph [10] plots sample cases as points defined by these two probabilities and is used to indicate which policies give good results. The Pareto plot shows a frontier facing the axes that gives the policies that are better than others in terms that there is no other policy that improves both the probabilities. This frontier defines policies that give the best compromises between the two criteria. It is then up to the user to choose which case on the Pareto frontier is best suited to a particular application.

To obtain sufficient accuracy in estimating the probabilities for the Pareto plot it was considered that the 40 years of data available would not be sufficient and thus it was desirable to generate additional typical data to test policies for the use of pipeline water.

For demonstration purposes a period of a thousand years was chosen. This allowed the different combinations of weather and storage that might occur to have a reasonable probability of being in the simulation sequence, and gave a sufficiently compact cloud of points on the Pareto optimum plots (Figures 7 and 8) to distinguish between the different policies. As seen in Figure 5 there is very little serial correlation between the rainfall in adjacent years. However the amount of water kept from one year to the next is important in determining when the dam will empty or overflow.

The effect and thus evaluation of the different control policies were examined using simulation. A monthly cycle was chosen for this simulation with the rainfall for the year determined by a random selection of a year's rainfall from the available records. In this manner it was possible to simulate a thousand years of dam operation and estimate the probabilities of the dam being either empty or overflowing.

Five different policies for the control of pipeline water were proposed and tested. The policies use the proportion α of the available pipeline water. The policies also use the current dam height h, the maximum dam height h_{max}, and a desired dam height h_{aim}. The policies tested were:

1 Take a constant proportion of the available pipeline water:

$$\alpha = \text{Constant} \tag{21}$$

2 Take pipeline water aiming to maintain about 70% full:

$$\alpha = \begin{cases} (h_{max} - h)/(h_{max} - h_{aim})/2 & h > h_{aim} \\ (h_{aim} - h)/h_{aim}/2 & h < h_{aim} \end{cases} \tag{22}$$

3 Take maximum pipeline water during dry season and none during wet season.

$$\alpha = \begin{cases} 0 & \text{Wet season (Dec Jan Feb)} \\ 1 & \text{Dry season} \end{cases} \tag{23}$$

4 Take maximum pipeline water if dam below 30%, take no pipeline water if dam above 70%, and a proportion of available pipeline water corresponding to proportion of dam contents between 30% and 70%

$$\alpha = \begin{cases} 0 & h > 0.7h_{max} \\ (h - 0.3h_{max})/(0.4h_{max}) & 0.3h_{max} < h < 0.7h_{max} \\ 1 & h < 0.3h_{max} \end{cases} \tag{24}$$

5 An on/off policy: no pipeline water if contents above V_{aim} and available pipeline water if below.

$$\alpha = \begin{cases} 0 & h > h_{aim} \\ 1 & h < h_{aim} \end{cases} \tag{25}$$

The simulations to evaluate these policies were actually carried out in a series of steps as indicated in the following subsections.

9.1. Determining good control parameters

The first step for each of the proposed policies was to determine good parameter values. For instance, in policy 5 it is necessary to determine the parameter for the storage volume V_{aim} that is used to trigger the use of pipeline water. Each of the policies has one or two parameters or constants that control the the way the policy operates.

For each policy multiple values of the parameters were selected for testing. For each set of parameter values several simulation runs of 1000 years were made and the number of months in which the dam was empty and full were recorded. These two values were plotted against each other on the Pareto optimisation graph as a single point. Over many repeated runs a cloud of points developed that showed the range of values typical of that control policy with the given parameter values.

Examination of the Pareto optimisation graph for a range of different parameter values in each of control policies allowed a choice to be made of good parameters for the policy. These parameter values were then used in the comparison of the different control policies.

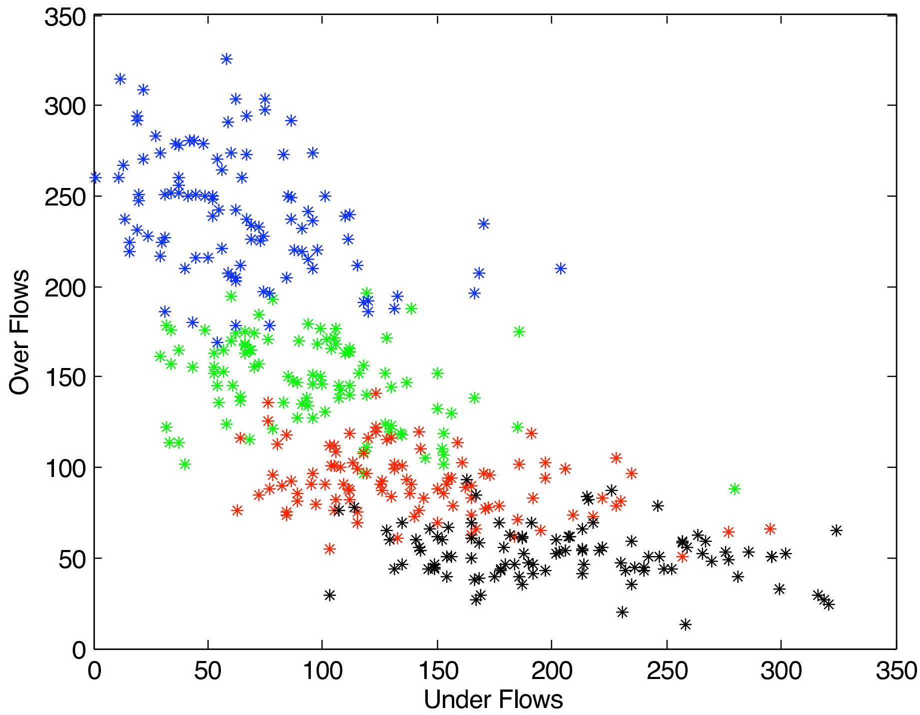

Figure 7. Effect of different parameter values for strategy #5 showing the number of months the dam over flows, plotted against the number of months the dam empties (under flows).

9.2. Determining a good control policy

The preceding section determined good control parameters for each of the control policies being considered. The next step is to apply the same method to determine which of the control policies give good performance. Again this was done using the the Pareto optimisation technique.

Each control policy was simulated 100 times over the period of 1000 years to determine the range of typical behaviour for the policy. Plotting these on the Pareto optimisation plot then determined which policies gave good performance and then a user could choose the policy that best suits the user's needs. It can be seen in Figure 8 that the 100 points plotted were sufficient to indicate the differences in performance of the control policies, without putting an excess of points onto the graph.

Figure 8 is the Pareto optimisation graph showing the evaluation of the of the five control policies. For operation with low risk of overflow policy #4 is the best, while for low risk of an empty dam policy #5 appears to be the best. Note that a different selection of parameter values can

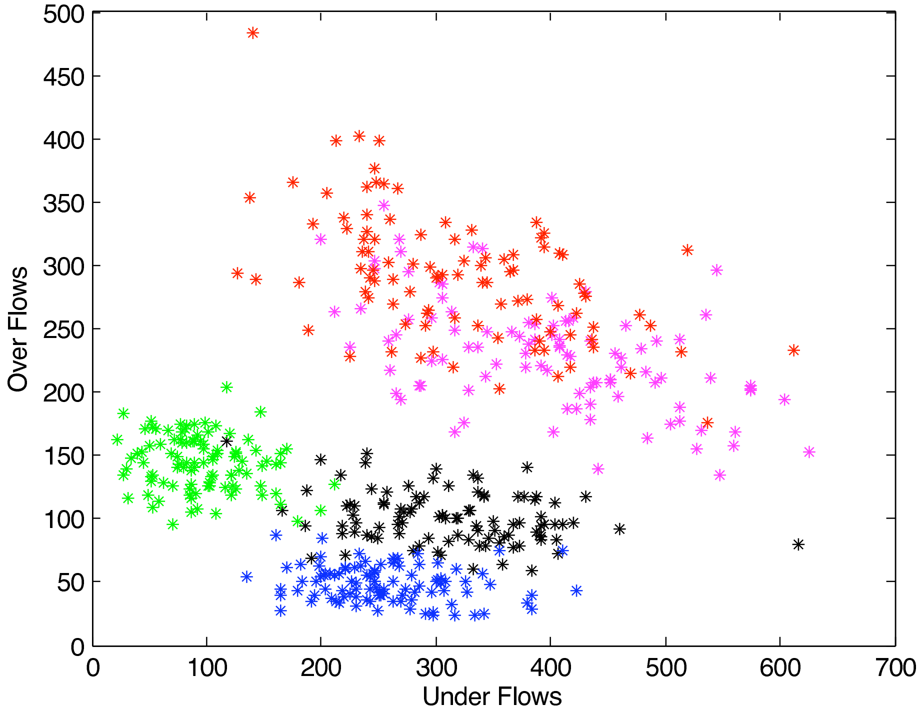

Figure 8. Comparison of control policies (Strategy #1 - Magenta, #2 - Black, #3 - Red, #4 - Blue, #5 - Green).

change the performance of the different strategies. In particular, a higher value of h_{aim} in policy #5 gives performance similar to, but not quite as good as, strategy #4. Strategies #1 and #3 give results significantly worse than the others, while strategy #2 is not as good as the two best strategies. Of the strategies considered, #5 is very simple and is either the best or close to the best. It may be possible to develop a policy that switches between aspects of strategies #4 and #5 that further reduces both objectives.

It should be noted that this is a demonstration of a technique that can be used to determine the best control policies. The MISG has concentrated on the method rather than the accuracy of the results. The method can be repeated using more accurate data and details specific to a particular mine site.

10. Further thoughts

There are three options that can be considered when determining a policy for the use of pipeline water:

The first case is when the pipeline can deliver sufficient water to satisfy the mine needs. For this case the mine should use dam water when it is available and use pipeline water otherwise.

The second case is when the rainfall can provide all the water the mine needs. In this case the dam will overflow and if it is sufficiently large there will be no need for pipeline water. If the dam is too small or the initial level of the dam is too low an addition of pipeline water will be needed, and this case becomes the same as the next case.

The final case is when both rainfall and pipeline water are needed to supply the mine. This is the case where control of the flow of pipeline water is needed. In this case, pipeline water is limited and not sufficient for the mine's instantaneous requirements (the first case covers when a limit is not relevant). Forward planning of pipeline use is needed to ensure that sufficient pipeline water is stored in the dam to cover the needs of the mine.

It is the last case that has interesting properties and these are examined in the next subsection.

10.1. Case where both rainwater and pipeline water are needed

There is significant variation in the rain, so a good policy for control of dam levels needs to take this into account. The balance of expected (i.e. mean) net usage can be determined to find the dam level (which can be negative). Next a safety margin to allow for the possible variation in the net water balance needs to be defined. Once these have been determined the amount of pipeline water needed, to cover both the expected water usage and the safety margin, can be calculated and the necessary time needed to deliver this amount determined. This amount of water is for the most pessimistic case.

The safety margin can be determined in different ways. An appropriate method is to determine the amount needed to reduce the probability of the dam going dry. As the more distant future is less certain the safety margin needs to be larger for more distant future times.

Given the expected water usage and the safety margin, the amount of pipeline water needed can be determined as follows:

1 Determine at each time that the water level drops below the safety margin, the amount of water needed to return the dam level to the safety margin value;

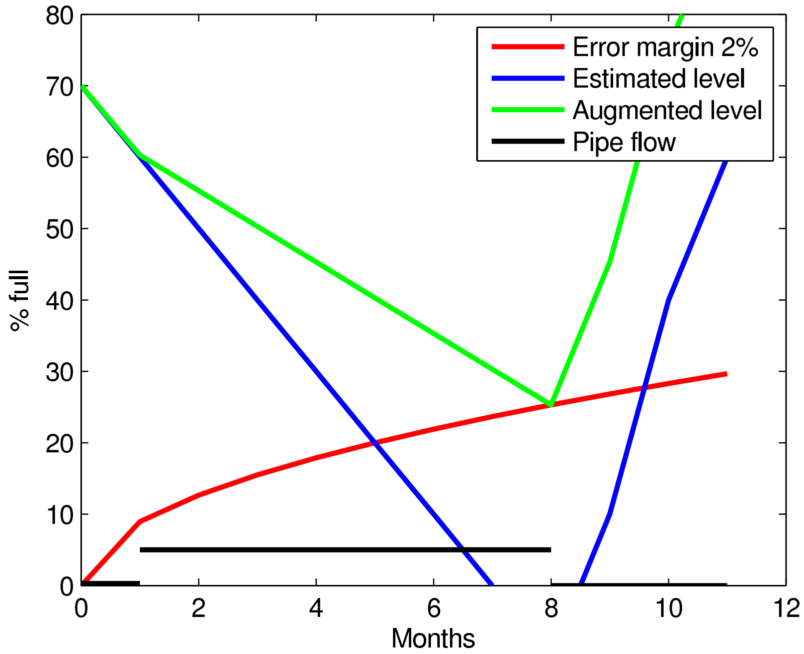

Figure 9. Initial prediction of water requirements.

2 Determine if the required amount of water determined in (1) can be obtained from the pipeline — if this amount of water cannot be supplied, then take the maximum amount of water available;

3 Taking into account the maximum flow from the pipeline, determine the latest time that water can be obtained from the pipeline to supply the amount determined in (1).

These steps determine the amounts of water needed to ensure the level does not drop below the safety margin in the worst case, as determined using an acceptably low probability for the low rainfall to occur. To protect against the possibility of the worst case the amount of pipeline water determined as needed during the next month should be obtained. However, typically the next month will not be a worst case and when the required pipeline water additions are recalculated at the end of the month they will usually be less than calculated in the previous step.

The required amount of margin can be estimated using the distribution of the variation in the net flows into the dam. This margin will increase as the time ahead increases. Figure 9 shows the initial pipeline water

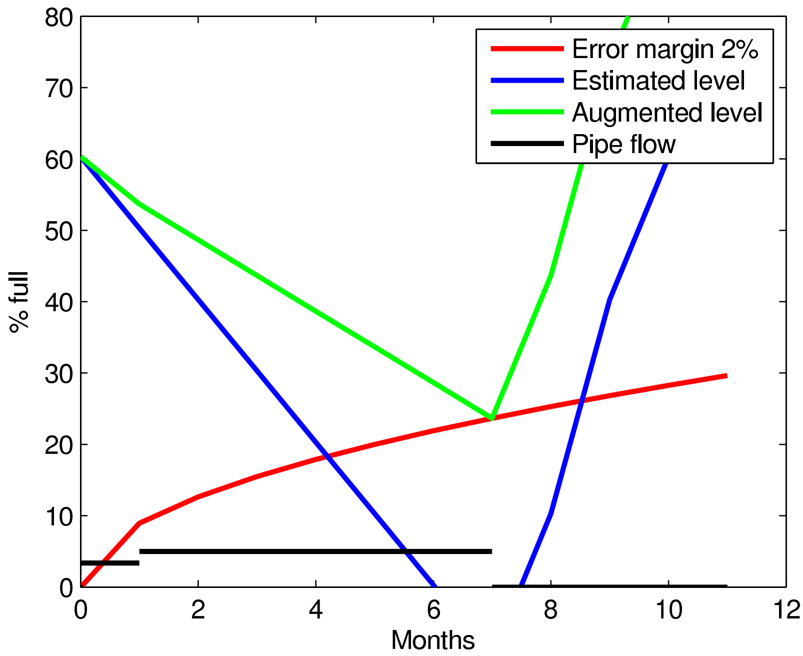

Figure 10. Prediction of water requirements after a one month step.

calculation. The red curve shows the increasing margin, and the blue the expected dam level without pipeline water, this goes negative indicating a need for additional water. The green line gives the dam level after water addition and the black line the pipeline water. Figure 10 gives a typical condition after the first time step. The deficiency in water is not as great as the worst case and the margins required for future times now being closer, are less, resulting in the water being required being less than previously estimated. Figure 11 shows a typical case of the progress of dam levels and the corresponding pipeline water flows, and Figure 12 show several alternative possible dam level trajectories. It can be seen in Figure 12 that only one of the trajectories reaches zero.

11. Conclusions and recommendations

An examination of the available data showed the variation in the rainfall dominates the analysis of the of the dam levels. The amount of rain in the wet season tends to follow a log Normal distribution and in the dry season there is little runoff into the dam. The monthly figures for

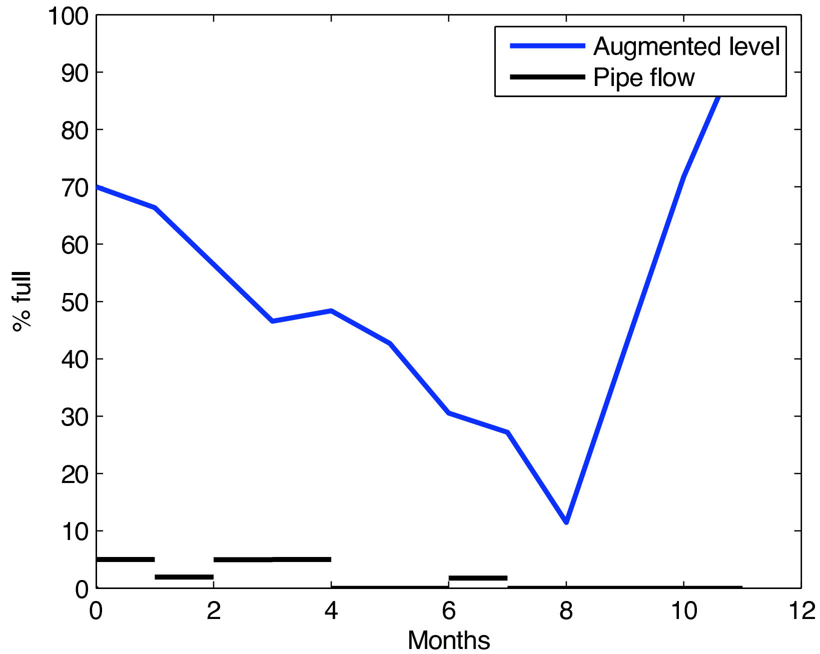

Figure 11. Typical progress in water consumption.

proportion of rainfall running into the dam seem a little simplistic and might be improved by a simple model of soil absorption.

This report has examined continuous and discrete analytical analyses. These use a distribution of the rainfall that does not closely match the actual distribution and do not easily take account of the changed conditions when the dam is empty. A simulation approach was found to be more useful for more detailed predictions.

A simulation based on wet and dry seasons demonstrated how the probabilities of the dam overflowing and of emptying can be estimated.

It was found that a feedback control that adjusts water inflow or outflow is needed to maintain an operation that has a low probability of running empty or overflowing.

Several simple control schemes were proposed and tested. A Pareto optimum plot avoids specifying a desired ratio of overflow time to empty time. A policy where pipeline water is used when ever the dam drops below a given level gave good results.

The case of most interest is where both rain water and pipeline water are needed. For this case a suitable policy looks forward to determine the

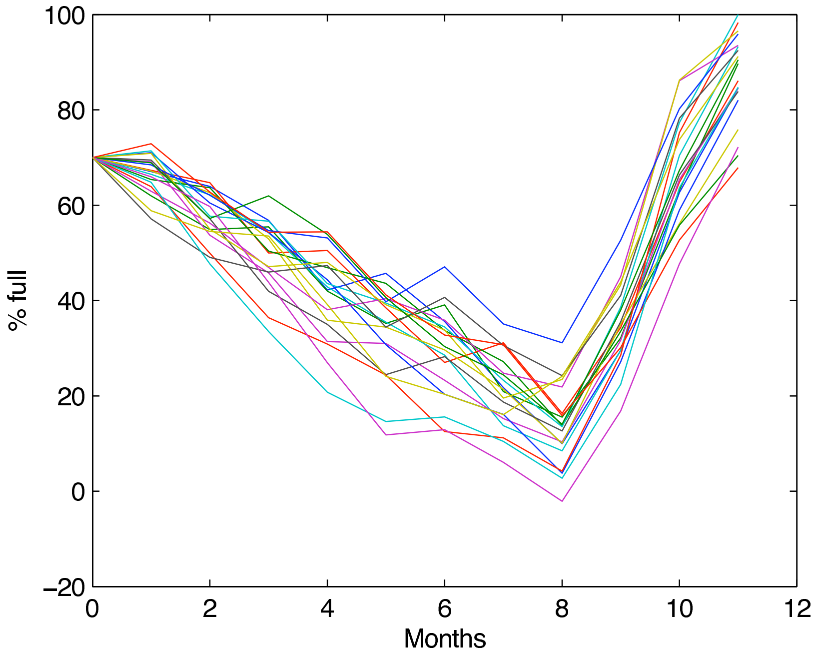

Figure 12. Alternative possible dam level sequences.

amount of water needed to reduce the probability of running empty to an acceptable level, and starts supplying pipeline water at the latest time that allows the required amount of water to be added. At regular time intervals, the amount of pipeline water needed is re-evaluated. Generally, when the situation is re-evaluated, it will be found that less water than was originally calculated will be needed, as the extreme case originally allowed for has not occured.

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