Segregation of a Two Species Granular Flow

Problem presented by Elkem a/s

Abstract

A cellular automaton model is presented for the segregation of a granular flow. The flow consists of particles of two different sizes, which in the specific industrial problem presented by Elkem are lumps of coal. It is known from experiments that these particles show different mobilities under different circumstances. This effect is incorporated in the current model via the inclusion of a 'hydrostatic pressure' term.

1 Introduction

Recently there has been rapidly increasing interest in the phenomenon of 'self organized criticality', wherein large interactive systems exhibit a predisposition to evolve toward a critical state where even a minor event can initiate a 'chain reaction' and lead to a major catastrophe (see, for example Per Bak & Kan Chen(1991)).

The paradigm for studies of self-organized criticality is the simple process of creating a pile of a granular material, for example sand, by dropping particles onto a solid surface. This problem was the subject of a careful experimental study by Held et al. (1990) which led to the development of various cellular automaton models. These include the model of Baxter & Behringer (1990) who endowed their sand particles with orientation. The more general problem of the dynamics of a pile of granular material was also studied by Fauve et al (1989) who noted that under the influence of vertical vibration, an initially planar layer of granular material formed a pile with a predictable surface angle.

In the present model, motivated by experimental observations, we pursue an alternative route and consider a granular flow that consists of two species of particles of different sizes. Our aim is to present a cellular automaton model which shows the segregation of the two types of particle as seen in experiments. This model will assume that the flow is two-dimensional, but can obviously be trivially extended to three dimensions.

The phenomenon of segregation has been studied with particular reference to the food industry by Barker & Grimson (1990). They noted the tendency of granular mixtures with two distinct grain sizes to segregate under vibration, the larger particles moving upwards to form a distinct layer from the smaller particles. The context of the present model however arises from the steel manufacturing industry, where granular mixtures of coal particles of two different sizes (the fuel for a smelting furnace) are poured into the top portion of an approximately rectangular hopper under gravity and drain through an exit hole in a bottom corner of the hopper.

Experiments show that, after the pile has developed in the hopper, a segregation phenomenon is observed where the larger particles form a vertical layer on the outside of the pile above the exit hole. Further towards the centre of the hopper above the exit hole, a vertical layer of small particles forms, so that the mixture which drains from the hopper is segregated. Under some circumstances, this effect can even be employed to provide an effective method of separating the two sorts of particles.

2 Experimental Observations and the Cellular Automaton Rules

In choosing the rules for our cellular automaton model we draw heavily from experimental observations. The work of Halvorsen (1991), in particular, gives an indication of the physical mechanisms causing the segregation. He highlights three phenomena of particular importance. These are:

(i) Flow occurs predominantly in a top layer near the surface of the growing pile. This layer is only a few particles deep and the particles within it experience large velocity gradients. The smaller, less flowable, particles separate downwards and join the slower flow in the centre of the heap while the larger particles flow rapidly along the surface.

(ii) In the internal flow there is a general percolation of the smaller particles through the relatively immobile larger particles.

(iii) In the interior flow the motion is governed more by the stresses experienced by the particles than by motion under gravity.

Typical pictures of cross-sections of a hopper show segregation resulting in the larger particles lying at or near the surface. There are large areas which contain mostly small particles and others in which the two sizes are of roughly equal proportion. Film of the filling of these hoppers also shows the rapid movement of the larger particles over the surface of the heap and the much slower flow of both sizes of particles in the interior.

In the model produced during the week of the study group we have ignored the effects of internal stresses and concentrated on the effects of the different mobilities of the two types of particle under different conditions within the pile.

By analogy with continuum models for fluid flows we introduce a 'hydrostatic pressure', defined at each point in the flow. In the present model, we take this to be the total weight of particles in the column immediately above that point. This pressure then determines which type of particle prefers to move down to a point below.

Our cellular automaton model starts with a square grid (the 'hopper') with n_r rows and n_c columns. Each of the squares in the grid (the 'sites') have unit volume. There is a general motion downwards (with rules to be described shortly) but there is no flow out of the side walls or out of the bottom except at designated exit holes. Particles are input over a given range of sites at the top of the array.

Definitions

(Input details)

i, j label of row, column

α volume of large particle

β volume of small particle $(0 < \beta < \alpha < 1)$

n_r, n_c number of rows and columns in hopper

(Calculational details)

M_{ij}, N_{ij} number of large, small particles in box (i,j)

$V_{ij} = 1 - \alpha M_{ij} - \beta N_{ij}$ volume of box (i,j) unfilled

$$P_{ij} = \frac{1}{n_r} \sum_{k=j+1}^{n^*_{i,j}} (\alpha M_{ik} + \beta N_{ik})$$

$$n^*_{i,j} = \min_{k \geq j+1} \{n_r\} \bigcup \{k : M_{ik} = N_{ik} = 0\}$$

R uniformly distributed random variable in [0,1] (chosen afresh each call)

S random integer -1,0,1 with respective probabilities 1/4, 1/2, 1/4 (chosen afresh each cal

Rules:

We begin with an empty hopper ($V_{ij} = 1$).

1) Add particles in prescribed number to the openings in the top row.

2) Move to bottom row and remove all particles (if any) from each exit site.

3) Move up one row and, unless this is now the top row (in which case return to 1)), order the sites in this row randomly.

4) Following this random ordering for the sites work along the row performing the following redistribution of particles, until the last site in this row has been dealt with in which case return to 3).

5) If $V_{ij} > \alpha$ (*i.e.* there is room for a large particle) and there are big or small available from the three sites above (immediately above and above diagonally) (or only two if the site is at the end of a row) then take one particle from site $(i+S, j+1)$ and place it in site (i, j) preferring to take a large particle if $P_{ij} < R$ and small otherwise. If the chosen site for removal does not contain such a particle (either it only contains the other size or contains no particle at all) then choose another of the three sites above, again at random. Repeat from 5) until $V_{ij} < \alpha$, then go to 6). If there are no particles in any of the three sites above return to 4), choosing a new site to receive more particles.

6) If $V_{ij} > \beta$ (*i.e.* there is room for a small particle) and there are small particles available from one or more of the three sites above, then take one small particle from site $(i + S, j + 1)$ and place it in site (i, j). If the chosen site does not contain a small particle then choose another of the three sites above. If there are no small particles available in any of the three sites above then return to 4), choosing a new site to receive more particles.

These are the rules governing the behaviour of the cellular automaton. We note several points of importance. First, the rows are dealt with in order, starting at the bottom and working to the top. This mimics the action of gravity in only filling a site after particles have left that site. Second, the particles are more likely to move downwards than diagonally but cannot move sideways. Third, near the surface, where the pressure term P_{ij} is smallest, the large particles prefer to move whereas in the interior where the pressure is greater the small particles are more mobile. Finally, unlike many cellular automata where the individual cells are updated simultaneously the present model uses a sequential updating with randomness playing a major role.

3 Results and Discussion

At this stage it is in order for us to mention the dimensionality of our model. Since the particles are constrained to move down or diagonally down we can adjust the 'angle of friction' for the flow by adjusting the aspect ratio of the hopper. Once that has been fixed the position of the entry and exit holes determines the rest of the geometry. The number of rows (and columns) can then be increased to improve detail. Finally, the volumes of the large and small particles (α and β) can be adjusted. In all there are thus four degrees of freedom for a given geometry.

In figs. 1, 2,3 and 5 we see results of the cellular automaton.

Figs. 1,2 and 3 show a hopper of dimensions 100 by 100 with inlet over the top left 30 and outlet over the bottom right 10. The particles have sizes 0.3 and 0.05. Each mark represents a site at which a large particle is present. Fig 1 shows the heap after 250 iterations and 2 and 3 after 500 and 1000 iterations respectively. Note the uniformity of the size distribution during the initial transient period and the curvature of the surface. As time progresses and the distribution settles down we see the appearance of segregation and the straightening of the free surface. This is shown schematically in fig. 4. First note that to the left of the line AB the flow is immobile (no motion can occur here since particles cannot leave from the bottom except at the right-hand end). Region I contains almost no large particles, and to the right of this region (in II) is a thin surface layer of large particles. Although the majority of the heap contains both large and small particles there is a region (III) where the larger particles dominate.

In fig. 5 we show a slightly different geometry. Here dimensions are as before but now the inlet is over sites 15 to 35 in the top and the outlets in sites 1 to 5 in the bottom row. Again the segregation can be very clearly seen.

These results are in qualitative agreement with experimental observations. To emphasise the importance of the pressure dependence in the present model we note that it is a very simple matter to 'switch off' this term in which case no segregation was ever detected.

4 References

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ADF, PW

250 iterations

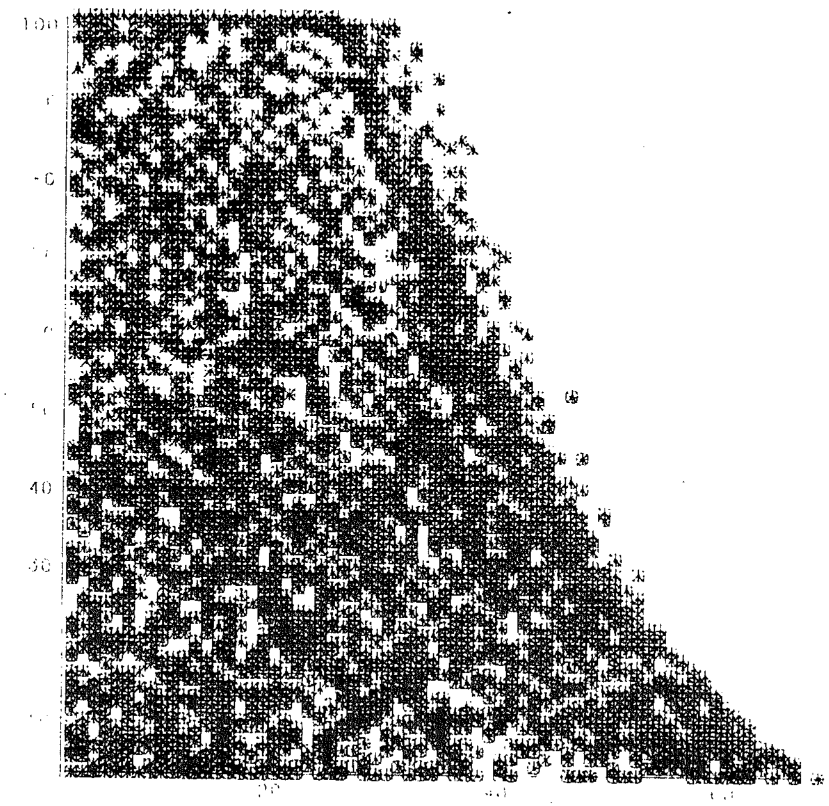

Fig 1

500 iterations

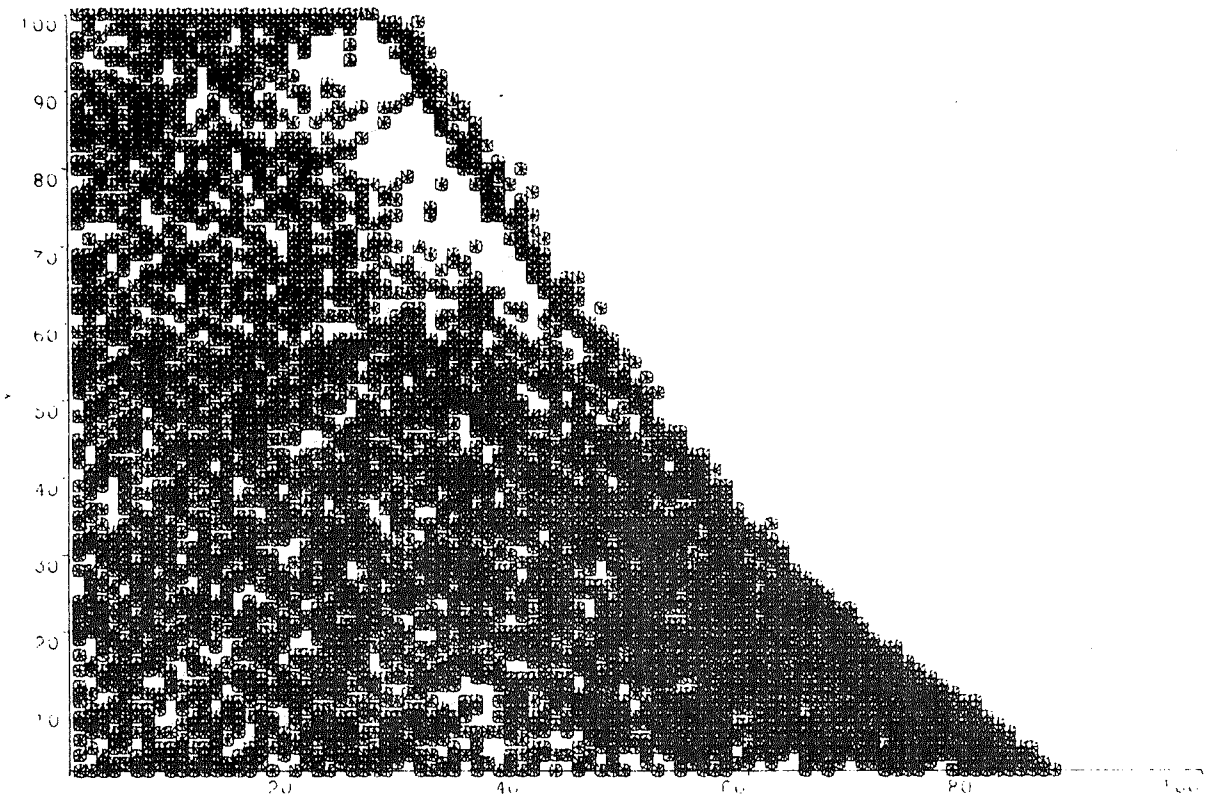

fig2

1000 iterations

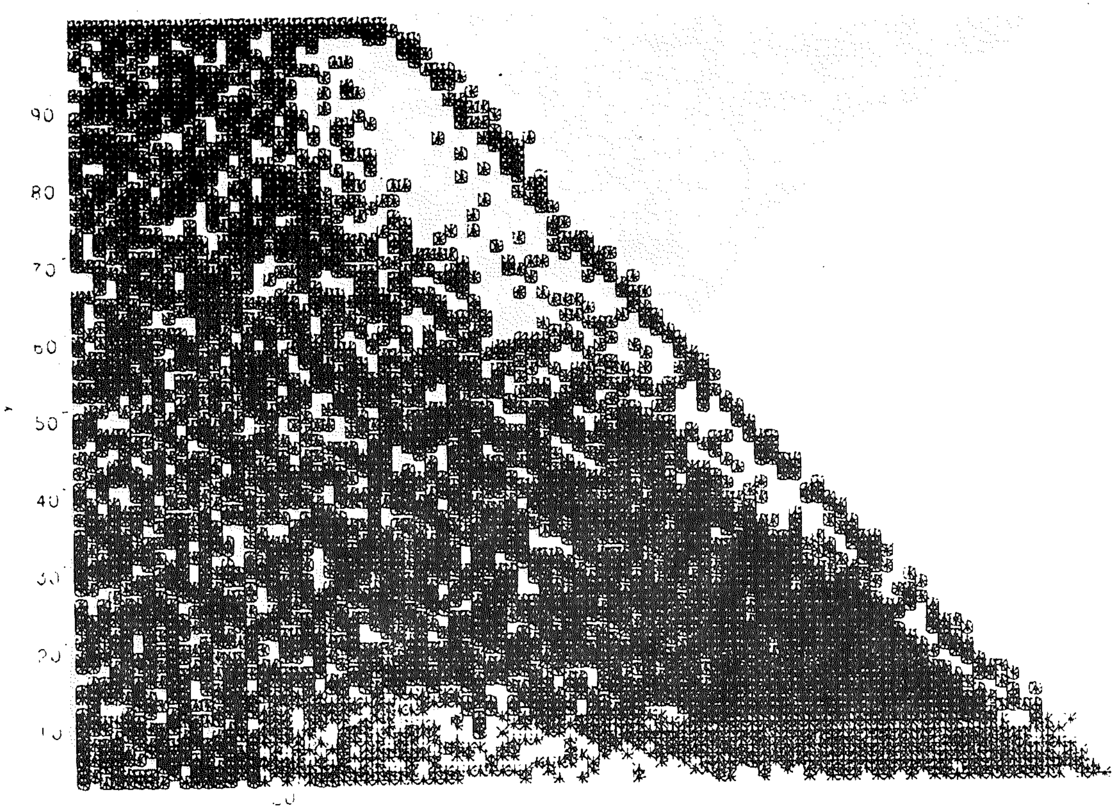

fig 3

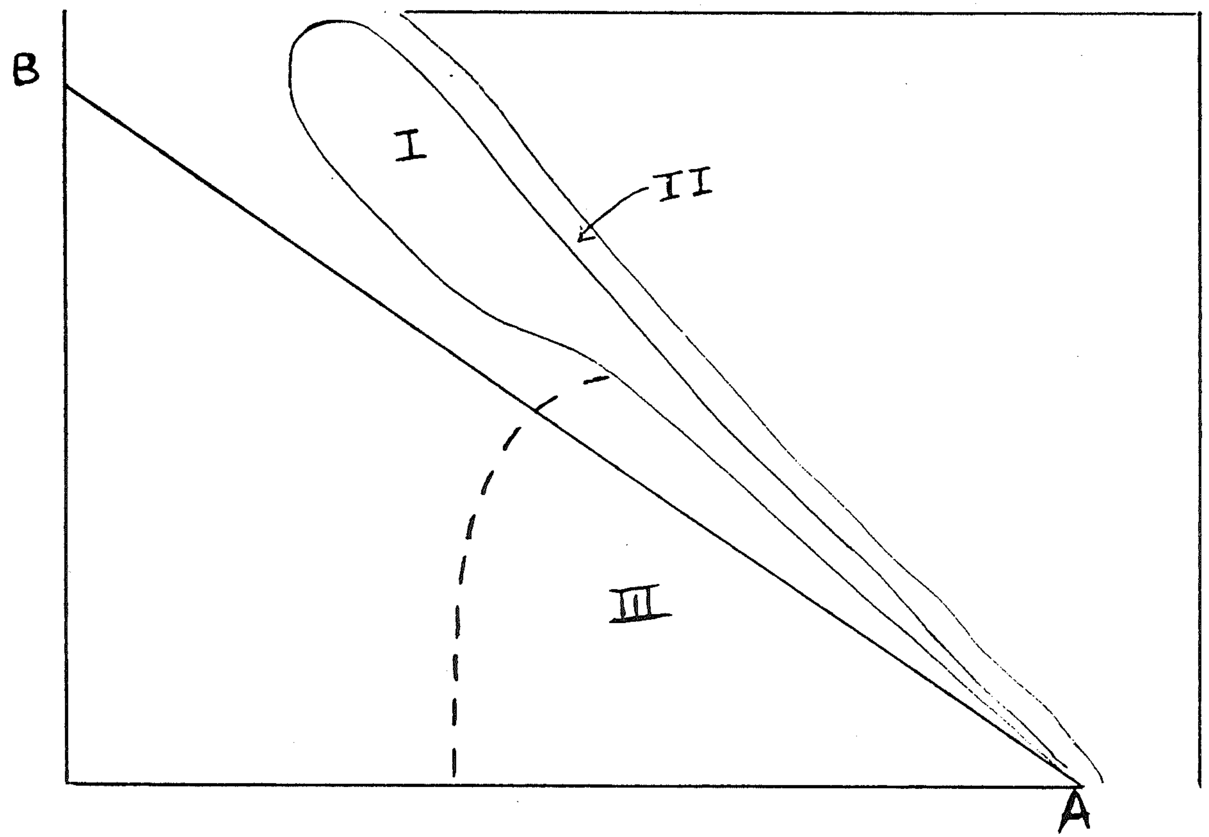

fig4

1000 iterations

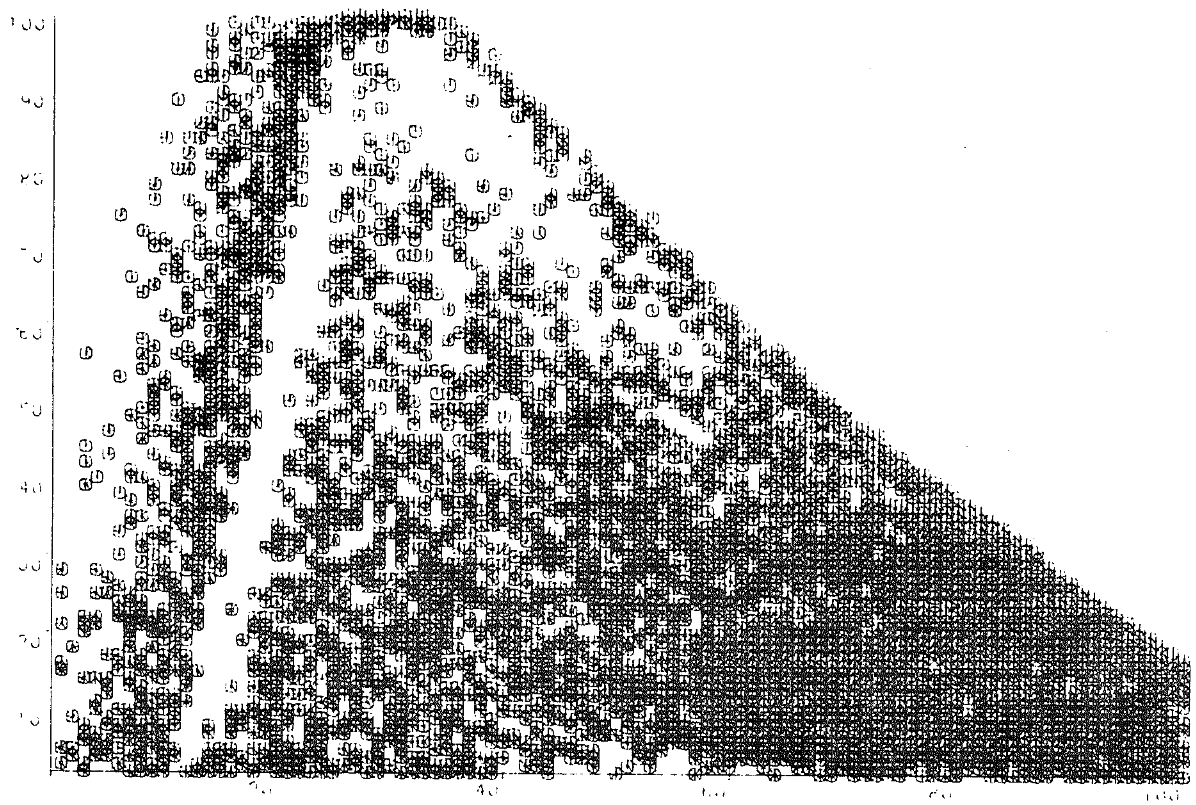

fig 5