

# MODELLING THE PHYSICS OF HIGH-SPEED WEIGHING

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## Abstract

Compac are interested in the weighing of fruit in a very short time, in an assembly-line situation. The fruit bounces and rocks in its weighing cradle, affecting the transient voltage output by the two load cells used to weigh the assembly. Compac find that their low-pass filter and averaging technique is not as accurate as they would like, for heavier fruit and shorter weighing times. In this report, we consider and solve simple models for harmonic motion, for bouncing and for rocking of fruit. We also consider beam-bending equations for the motion of a load cell, and power spectra of fruit weighing data produced by Compac. Some consideration is given to using the data to fit critical parameters for the load cells, which govern how they vibrate when loaded with fruit. We find that the bouncing (and not the rocking) of fruit is the likely cause of the lower frequency oscillations that affect accuracy for heavier fruit and/or faster speeds.

## 1. Introduction

Compac Sorting Equipment Auckland (Compac) is a company that manufactures and exports high-speed, accurate sorting systems for fruit and vegetables. Their sizers operate at 10-15 pieces of fruit per second per lane. Each piece of fruit is weighed separately, in less than 1/10 of a second. Compac require a mathematical model of the weighing process, that will help to improve accuracy of weighing heavier fruit (more than 250g) at higher speeds (in less than a tenth of a second).

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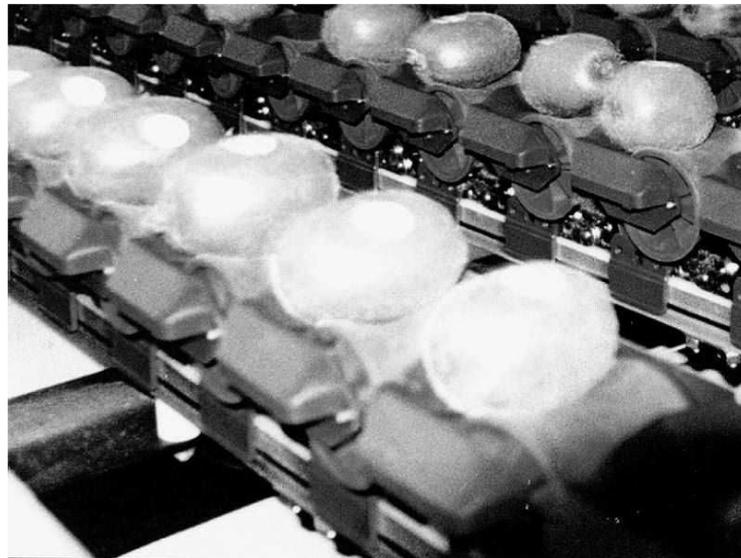
Table 1. Nomenclature

symbol	definition	units
$a$	load cell plate width	m
$b$	a parameter in $J$	-
$A$	beam cross-sectional area	m <sup>2</sup>
$E$	beam Young's modulus	Pa
$F$	applied force	kg.m.s <sup>-2</sup>
$I$	beam moment of inertia	kg.m <sup>2</sup>
$J$	moment of inertia	kg.m <sup>2</sup>
$k$	spring constant for a load cell	kg.s <sup>-2</sup>
$M$	total mass of fruit, carrier and load cell	kg
$R$	amplitude	m
$T$	distance between load cells	m
$x_1$	displacement in load cell 1	m
$x_2$	displacement in load cell 2	m
$y$	beam displacement	m
$\lambda$	decay constant	s <sup>-1</sup>
$\nu$	effective damping in a load cell	kg.s <sup>-1</sup>
$\omega$	frequency	s <sup>-1</sup>
$\rho$	beam density	kg . m <sup>-3</sup>
$\theta$	$(x_1 - x_2)/T$	-

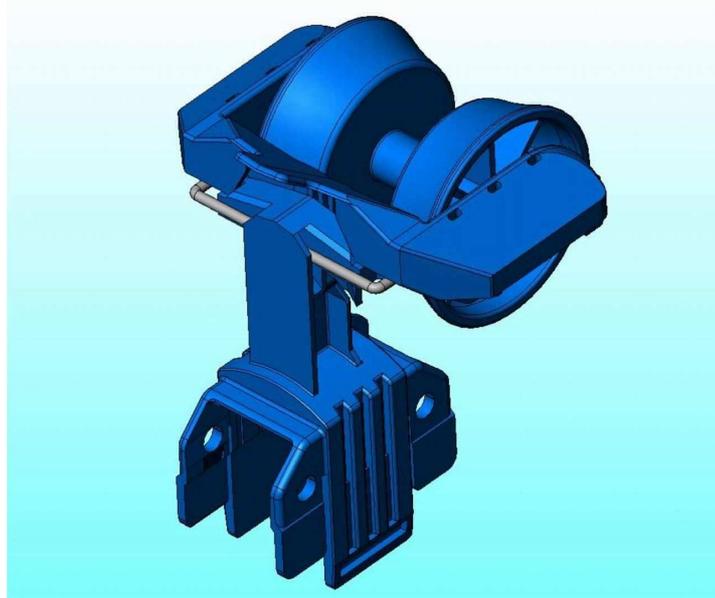
They also asked for help with reducing the size and inherent instability of the weighing assembly — it would be a lot easier and quicker if it could be integrated into the system that pulls the fruit along on a chain, rather than the most successful present setup, which has a separate weighing table bolted securely to the floor, and carefully aligned with the fruit track.

Each fruit is carried separately (Figure (1)), and weighed along with its carrier by passing it over two load cells. The carrier (Figure (2)) has four contact points. Two points on each side of the carrier slide along a steel plate mounted on a load cell. The load cell is cantilevered, is typically rated to 6kg, and is sensitive to shear rather than bending. Figure (3) is a picture of one load cell with an carrier moving over it (hand-held). A load cell contains resistances which change under compression/tension, arranged in a Wheatstone Bridge. The spacing between fruit containers is such that about 100mm is needed for each fruit, and 10 fruit/second corresponds to a speed of about 1 m/s. The voltages are sampled at 4 kHz using a 12 bit ADC.

In the existing approach, the signal from each loadcell is amplified and low-pass filtered. The filter is a fifth-order Butterworth filter set at about 55 Hz. The tail end of the signal is averaged, to obtain a



*Figure 1.* Kiwifruit in their containers. Movement is right to left. Each container can rotate fruit, tip fruit out, and float free of the chain (vertically) while being weighed.



*Figure 2.* Detailed drawing of a fruit carrier. Towing direction is right to left. The top part floats independently of the lower part, when on the loadcell.



*Figure 3.* A loadcell with the steel plate on top, and a (handheld) carrier moving over it from right to left. Note the two tracks worn into the steel plates by the moving contact points of the carrier.

mass that is required to be accurate to less than 1g. Empty containers are weighed initially, and (during fruit sorting) whenever they randomly happen to be empty. Each container is tracked individually, as their weights differ. The method also corrects for any drift in the tare weight of each container during a sorting run.

## 2. Data Analysis

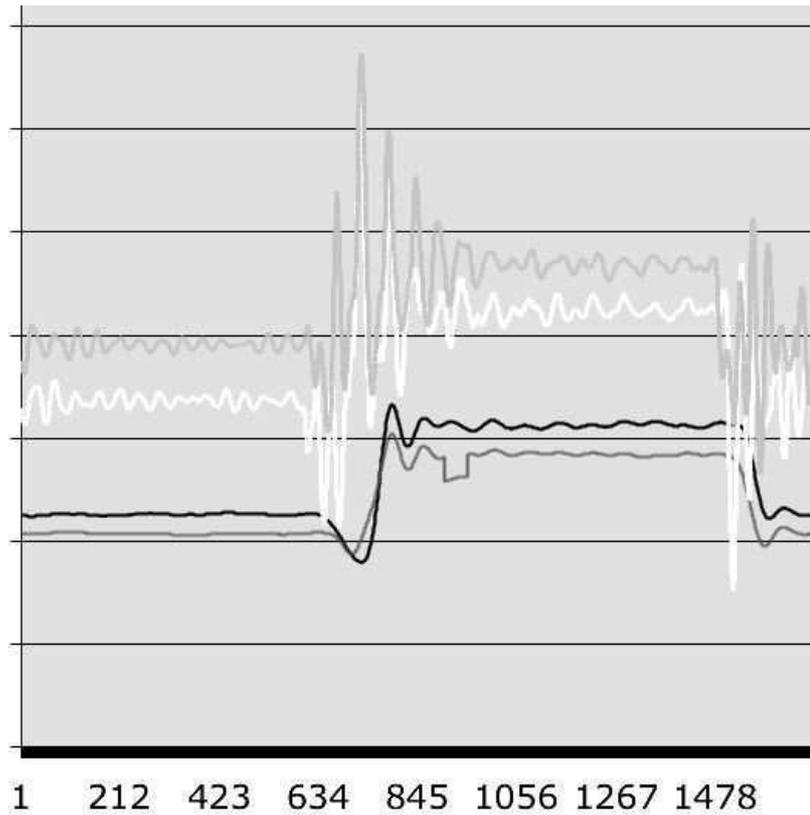
Compac provided data from a number of tests, showing filtered and unfiltered voltages from a pair of load cells for a variety of fruit of known weight, travelling at various speeds. A large effort went into examining this data, and the power spectra (see section (4)), to see what were the dominant frequencies and how these frequencies changed with speed and mass.

Figure (4) shows a typical set of filtered and unfiltered voltages from a pair of load cells, for a 200g weight at 0.5m/s. Note that the low-pass filter is successful in removing most of the oscillation, so that averaging the last part of the filtered signals is found to give an accurate representation of the weight of fruit and carrier.

In comparison, Figure (5) shows the voltages for a rubber “orange” weighing 513.1g travelling at the same speed (0.5ms). There is more oscillation in the filtered signals, which causes larger errors in measured weights. When fruit goes faster, the main problem is that the signal time is shorter, so that there is less time for the transients to decay to a steady voltage.

## 3. Springs and Rocking

Some existing studies [2, 3, 4] show that when the motion is that of a simple harmonic oscillator, it is possible to filter out the transient oscillation and find the steady state (the total mass) very rapidly. These adaptive filters essentially use the characteristics of the load cell as a simple harmonic oscillator (frequency, damping, effective mass), to rapidly find the added mass. For the cases presented [2, 3, 4], it is found that an accurate mass is obtained within just one half cycle of the oscillator. Besides its speed, an adaptive filter can handle a wide variety of masses, unlike a fixed filter, which does not compensate for changes in natural frequency with mass. This looks like a tempting approach for the Compac problem, but as will be seen in the section on power spectra, the oscillations observed in the Compac data have several important component frequencies, not just one. This suggests that a simple harmonic oscillator might not be an accurate enough model here.



*Figure 4.* Voltages from the two loadcells, as a 200g weight and its container pass over at 5 fruit/second (0.5m/s). The weight is responsible for the central part of the signal, between the values 634 and 1478. Also visible are the voltages from empty containers before and after the weighted one. There is very little time between one container leaving the loadcell and the next one coming onto it. The two larger signals are the raw data, and the two lower amplitude signals are the outputs from the analogue low-pass filters used by Compac. Some offsets in voltages and times have been introduced to more clearly view the signals.

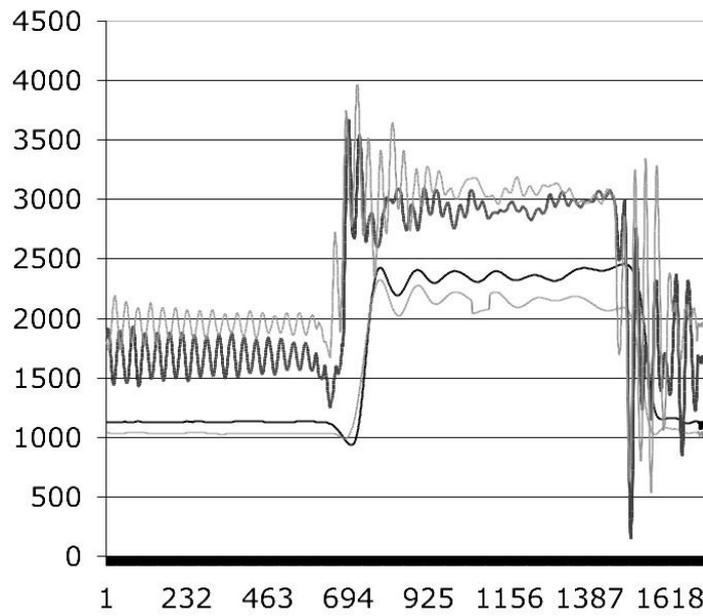


Figure 5. Voltages from the two loadcells, as a 513g imitation orange and its container cross at 5 fruit/second (0.5m/s). The two larger signals are the raw data, and the two lower amplitude signals are the outputs from the analogue low-pass filters used by Compac.

We first considered simple models of the motion of a load cell when a weight is placed on it, and of the possible rocking motion of the fruit and carrier. A model which couples these two motions is presented here. The fruit and carrier (treated as one mass) are allowed to rock sideways from one load cell to the other, and to move up and down on the load cells. It is assumed that the carrier is always in contact with the top of a load cell. The effect of horizontal velocity along the top plate attached to the load cell is ignored here — model equations for this are presented in a later section.

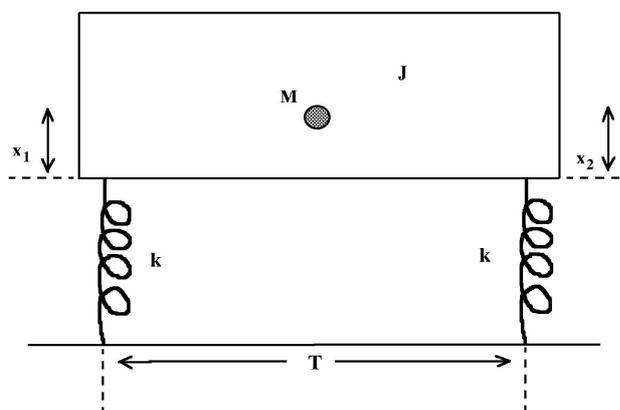


Figure 6. A sketch of our simple model, incorporating up-and-down motion plus rocking from side to side.

Then defining

$$x = (x_1 + x_2)/2$$

where  $x_1$  and  $x_2$  are the displacements about equilibrium in the two load cells, and

$$\theta = (x_2 - x_1)/T ,$$

where  $T$  is the distance between loadcells, equating the forces acting on the system gives the simple damped harmonic motion

$$M\ddot{x} + 2\nu\dot{x} + 2kx = 0 , \quad (1)$$

where  $M$  is the total mass of fruit plus carrier plus the effective mass of the load cells,  $\nu$  is the effective damping in each loadcell, and  $k$  is the

effective spring constant of each of the loadcells. Equating moments of inertia about the midpoint at  $T/2$  gives

$$2J\ddot{\theta} + T^2\nu\dot{\theta} + T^2k\theta = 0, \quad (2)$$

where  $J = \int r^2 dm$  is the moment of inertia of the system about the midpoint. Note that the choice of coordinate system has decoupled the motion — the up-and-down motion is represented by  $x$ , and can be solved independently of the rocking motion represented by  $\theta$ .

Initial conditions, if the fruit container with extra mass  $m$  was to arrive in isolation, would be that  $x(0) = mg/(2k)$ , since the displacement  $x$  from equilibrium is zero when the extra weight  $m/2$  of fruit plus container on each loadcell has stabilised (and static force  $kx$  matches  $mg/2$ ), and  $\dot{x}(0) = 0$ . However, in practice the loadcells are still bouncing from the previous container leaving them, so initial conditions will only approximate these values to some degree.

The general solutions to eqns. (1) and (2) take the form of decaying oscillations  $\exp(\lambda t)$ , where for  $x$ ,

$$\lambda_x = -\frac{\nu}{M} \left( 1 \pm \sqrt{1 - \frac{2kM}{\nu^2}} \right),$$

and for  $\theta$ ,

$$\lambda_\theta = -\frac{T^2\nu}{4J} \left( 1 \pm \sqrt{1 - \frac{8kJ}{T^2\nu^2}} \right).$$

For up-and-down motion  $x$ , the frequency of oscillation is the imaginary part of  $\lambda_x$  (in radians/s),

$$\omega_x = \sqrt{\frac{2k}{M} - \frac{\nu^2}{M^2}}, \quad (3)$$

and for rocking motion  $\theta$ , it is

$$\omega_\theta = \sqrt{\frac{T^2k}{2J} - \frac{T^4\nu^2}{16J^2}}. \quad (4)$$

Both frequencies decrease for heavier fruit (larger  $M$  and  $J$ ). The damping rates also decrease even more dramatically as the total mass increases.

A frequency of zero corresponds to critical damping, and an imaginary value for a frequency means the system is overdamped. In both cases there is no oscillation, just exponential decay.

Now the moment of inertia  $J$  about the centre is written in the form  $MT^2/b^2$ , where  $b$  parametrises the moment of inertia. A symmetric load with centre of mass in the centre, then corresponds to  $b \geq 2$ , with  $b = 2$  if half of  $M$  is directly over each load cell, and  $b \rightarrow \infty$  as  $M$  concentrates at the midpoint.

A sphere of uniform density and mass  $M$  has  $J = 2MR^2/5$  where  $R$  is the radius of the sphere. If the mass of the carrier is ignored, then  $b^2 = \frac{5}{2} \frac{T^2}{R^2}$ , and if the fruit has a diameter roughly the same as the width of the carrier  $T$ , then  $b^2 \approx 10$ .

Retaining a general dependence on  $b$ , it follows that

$$\frac{\omega_\theta^2}{\omega_x^2} = \frac{b^2}{4} \left( \frac{\frac{2k}{M} - \frac{\nu^2}{M^2} \frac{b^2}{4}}{\frac{2k}{M} - \frac{\nu^2}{M^2}} \right)$$

As the damping  $\nu$  tends to zero, this ratio tends to  $b^2/4 \geq 1$ . The two frequencies are equal if half of the load mass is concentrated directly over each load cell. Otherwise, the frequency of rocking is larger.

For nonzero damping  $\nu$ , the frequency ratio first increases above 1, then decreases to zero, as  $b$  increases. That is, for sufficiently damped motion, it is possible that the rocking frequency is less than the up-and-down frequency.

#### 4. Power Spectra

A number of power spectra of the Compac data were examined, for the different fruit weights and speeds. In Figs. (7) & (8) are shown computed spectra for 15 datasets at 300cpm and 10 datasets at 600cpm, for two different fruit masses. For each dataset, only a subset of the (unfiltered) data is used - 512 data points starting at the 800th data point for the 300cpm cases and 256 points starting at the 400th data point for the 600cpm cases. In this way the spectra are only for the loaded cups. Similar plots were made for the other weights. Observations that can be made from the graphs are that

- There are two, or sometimes three, dominant frequencies in each of the spectra.
- There is virtually always a frequency around 120Hz, this being the higher of the two frequencies.
- The lower frequency generally decreases with increasing weight of fruit, and its amplitude or relative importance generally increases with increasing weight.

- There appears to be little dependence of the dominant frequencies on line speed, although there is clearly more noise at the higher line speed.
- There is a reasonable amount of scatter between replicates of the same fruit and line speed.

The problem for Compac is the lowest frequency, which gets past their low-pass filter for heavier fruit. Higher speeds do compound the problem, as the shorter times mean that damping has less chance to reduce amplitudes. A low-pass filter also has a response time, before the filtered signal gets close to its asymptotic (steady) value. This response time can become critical for faster speeds. We noted that the lower frequency behaviour, as observed in the filtered data, appears to be in phase between the two load cells, hence corresponding to up-and-down motion rather than rocking. The high frequency appears typically to be a rocking motion, out of phase between the two loadcells.

A puzzle for us is that the high frequency is apparently almost independent of mass. It is seen in all data, including data with no fruit in the container. This contradicts our simple model, which predicts that frequencies will reduce as mass is increased. It would be consistent with a rocking motion due to the moment of inertia of the empty carrier, rocking independently of the fruit it is carrying.

Another puzzle is the extra frequency that is sometimes seen in spectra. It is suggestive of coupled oscillators, perhaps the carriers flexing under the weight of the fruit being the extra oscillator. Coupled oscillators can exhibit a range of interesting phenomena, including phase locking (where only one frequency is seen, which may be between the natural frequencies of the separate oscillators), and period-doubling bifurcations to chaotic behaviour. The first step in the period-doubling sequence will give two frequencies, close to the original frequency.

## 5. Filtered Data

We digitally filtered the data from one load cell for a 403.5g imitation lemon, using a second-order Butterworth (low-pass) filter at various cutoff frequencies, and compared with the Compac analogue filtered data. Figure (9) shows the results.

Compac's filter appears to be set at about 60Hz, which may be a little higher than desirable. The filtered signal from our 30Hz filter looks reasonably steady after 1/20s. This may be too slow for faster fruit speeds, however. The lower the filter cutoff frequency, the slower the filtered signal is to stabilise.

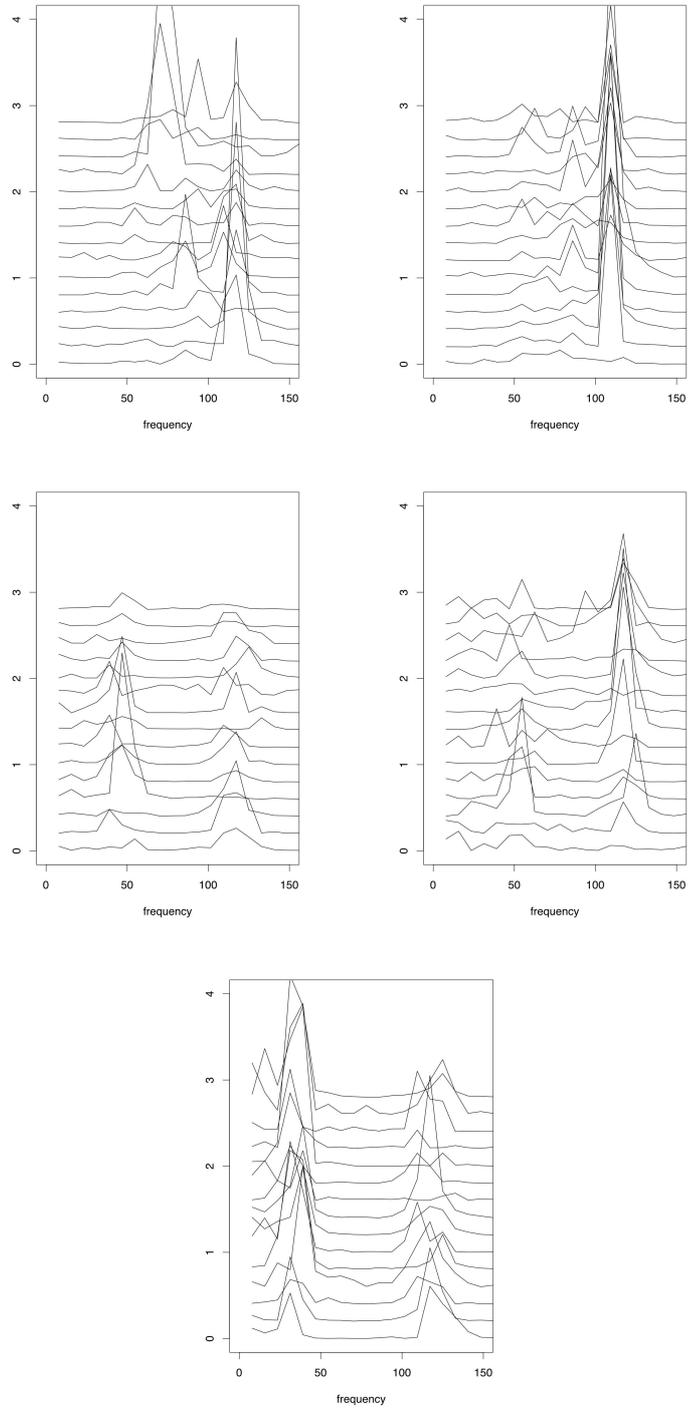


Figure 7. Frequency spectra for 5 different standard fruit weights (137.2g, 200g, 293g, 403.5g and 573g) at a line speed of 300cpm.

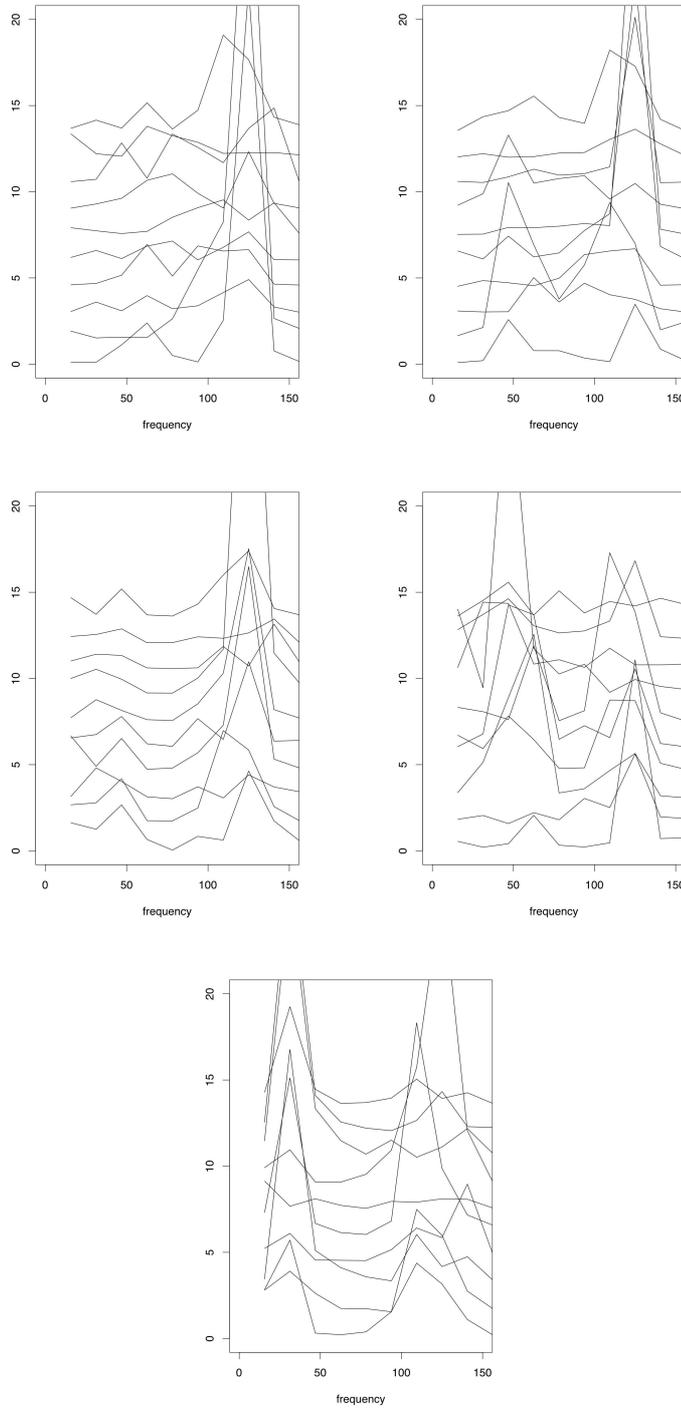
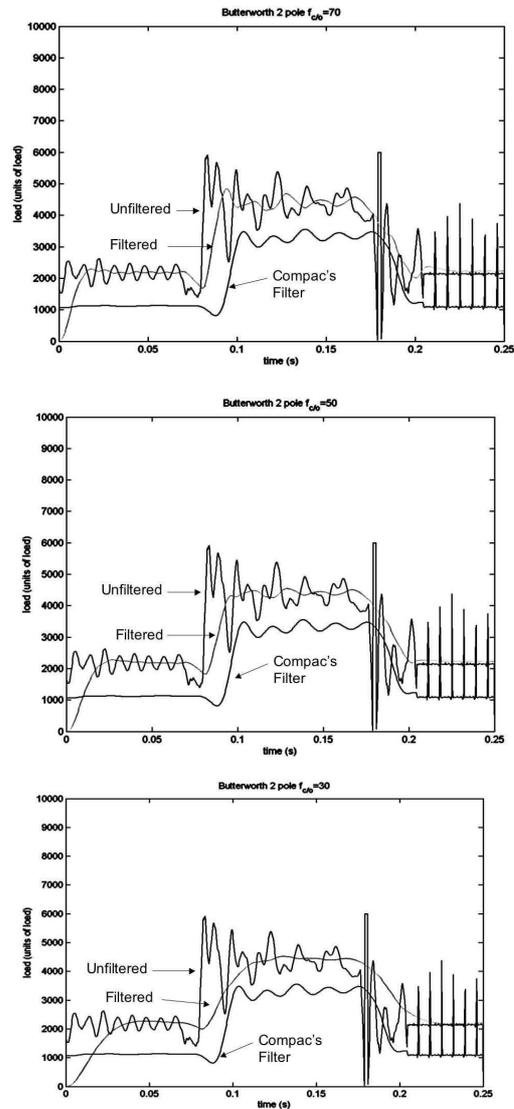


Figure 8. Frequency spectra for 5 different standard fruit weights (137.2g, 200g, 293g, 403.5g and 573g) at a line speed of 600cpm.



*Figure 9.* A graph showing raw data from one load cell, this data digitally filtered by our own low-pass filter, and Compac's analogue filtered data. The fruit was a 403.5g lemon, and the digital filter is set at 70Hz, 50Hz and 30 Hz in the first, second and third plots respectively. (i) The estimated weight using the unfiltered data; (ii) The estimated weight using the filtered data; (iii) The estimated weight using the Compac's filtered data.

## 6. Load-cells are Beams

We considered the cantilevered load-cells more carefully, since they are beams rather than springs. With a forcing  $F(t)$  on the end  $x = L$  of a solid cantilevered beam, and vertical displacement  $y$ , then if we ignore damping, a force balance gives the standard beam-deflection equation

$$\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = 0, \quad (5)$$

where  $\rho$  is the density of the beam,  $A$  the cross-sectional area,  $E$  the Young's modulus, and  $I$  the moment of inertia about the point where it is secured  $x = 0$ . Boundary conditions are

$$\begin{aligned} y(0) = 0, & \quad \frac{\partial^2 y}{\partial x^2}(L) = 0, \\ \frac{\partial y}{\partial x}(0) = 0, & \quad \frac{\partial^3 y}{\partial x^3}(L) = \frac{F}{EI}. \end{aligned}$$

Separating variables with  $y = p(t)r(x)$  gives

$$\frac{d^2 p}{dt^2} + \omega^2 p(t) = 0, \quad p(0) = p_0, \quad p'(0) = \dot{p}_0, \quad (6)$$

and

$$\frac{d^4 r}{dx^4} - \beta^4 r(x) = 0, \quad r(0) = 0, \quad r'(0) = 0, \quad r''(L) = 0, \quad r'''(L) = \frac{F}{EI p(L)}, \quad (7)$$

where

$$\beta^4 = \frac{\rho A \omega^2}{EI}.$$

The equation for  $p$  is equivalent to that for simple harmonic motion, with effective mass and spring constant:

$$m \equiv \rho AL, \quad k \equiv EI\beta^4 L.$$

For  $F = 0$ , the first few eigenvalues for  $\beta$  are

$$\beta_1 \approx 1.88/L, \quad \beta_2 \approx 4.7/L, \quad \beta_3 \approx 7.9/L,$$

so that  $\omega$ , the frequency of oscillation of the beam, varies as  $L^{-2}$ .

This would make the load cell very sensitive to the position of the load, if the load were to travel along the beam that is the load cell. But the load travels along a plate, that is attached to the end of the load cell, as illustrated in Figure (10).

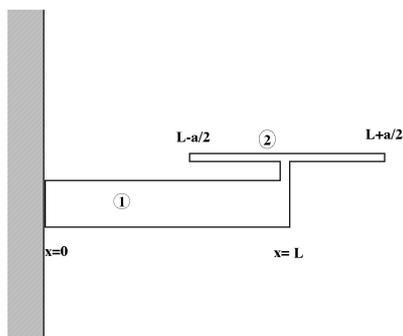


Figure 10. A sketch of load cell (1) plus steel plate (2). The fruit carrier slides along the top plate, from the rear to the front, at roughly constant speed, while it is being weighed.

The design means that the load delivers a varying torque to the end of the load cell. The load cells are designed to be sensitive to shear rather than bending, minimising the effect of this varying torque.

We write down the undamped model equations here, for completeness.

For the load cell (1),

$$\rho_1 A_1 \frac{\partial^2 y_1}{\partial t^2} + E_1 I_1 \frac{\partial^4 y_1}{\partial x^4} = 0, 0 < x < L,$$

and for the plate (2)

$$\rho_2 A_2 \frac{\partial^2 y_2}{\partial t^2} + E_2 I_2 \frac{\partial^4 y_2}{\partial x^4} = F(x, t), L - a/2 < x < L, L < x < L + a/2.$$

Boundary conditions are,

$$x = 0: y_1 = \frac{\partial y_1}{\partial x} = 0,$$

$$x = L \pm a/2: \frac{\partial^2 y_2}{\partial x^2} = \frac{\partial^3 y_2}{\partial x^3} = 0,$$

and at  $x = L$ , there is a force  $f(t)$  and a moment  $m(t)$  on the loadcell, so that

$$y_1 = y_2, \quad y_1' = y_2',$$

$$E_1 I_1 \frac{\partial^2 y_1}{\partial x^2} = m(t), \quad \left[ E_2 I_2 \frac{\partial^2 y_2}{\partial x^2} \right]_{L^-}^{L^+} = -m(t),$$

$$E_1 I_1 \frac{\partial^3 y_1}{\partial x^3} = f(t) \quad , \quad \left[ E_2 I_2 \frac{\partial^3 y_2}{\partial x^3} \right]_{L^-}^{L^+} = -f(t) .$$

The square brackets indicate the jump in the value inside, across  $x = L$ .

### 6.1. Fitting Beam Parameters

A preliminary attempt was made to use the up-and-down model results together with the data from Compac, to find the effective mass, damping and spring constant for a load cell. The filtered data is used, as this appears to have the rocking motion removed. The expressions to be used are, frequency

$$\omega_x = \sqrt{\frac{2k}{M} - \frac{\nu^2}{M^2}} \quad ,$$

and damping

$$\nu = - \left( \frac{M}{t_1 - t_2} \right) \ln \left[ \frac{y(t_1)}{y(t_2)} \right] \quad ,$$

where  $t_1$  and  $t_2$  are two successive times at which the voltage is a maximum (same phase). Another equation is needed to find the third parameter, and the amplitude of oscillation is the remaining unused property of the signal.

Two methods for using amplitude are outlined here, the first assumes that there is a constant forcing of the oscillator at frequency  $\omega_f$  which is responsible for a persistent signal after transients have died away. If the amplitude of oscillation with weight  $m_1$  on the loadcell is  $R_1$ , and with weight  $m_2$  is  $R_2$ , then

$$\frac{R_1^2}{R_2^2} = \frac{\left( k - m_2 \omega_f^2 \right)^2 + \nu^2 \omega_f^2}{\left( k - m_1 \omega_f^2 \right)^2 + \nu^2 \omega_f^2} .$$

The second method assumes zero initial voltage and zero rate of change of initial voltage, before an extra (known calibration) mass  $m^*$  moves onto the loadcell, and the loadcell is again modelled as a simple harmonic oscillator. Then ignoring damping, the output from the loadcell is

$$x = \frac{m^* g}{k} [1 - \cos(\omega t)]$$

so that the first peak in  $x$  has height  $2m^*g/k$ . We know the added calibration mass  $m^*$ , so the first peak gives us  $k$ .

Some preliminary calculations using the first of these two amplitude methods on the data provided by Compac suggest that

$$k \approx 8000\text{kg/s}^2, \quad m_{\text{eff}} \approx 60\text{g}, \quad \nu \approx 0.4\text{kg/s},$$

where  $m_{\text{eff}}$  is the effective mass of just the loadcell. These numbers are not very accurately determined at present, but they do compare with the values listed in [3] (for a different loadcell),  $k = 2700\text{Pa}$ , effective mass  $500\text{g}$ , an damping factor  $\nu = 3.5\text{kg/s}$ .

Then if  $a = 6$ , assuming fruit and carrier rock as one, and the total mass (fruit and carrier and load cells) with a  $200\text{g}$  fruit added is

$$M = 200 + 122 + 60 + 60 \approx 450\text{g},$$

the damping terms in expressions (3) and (4) for the frequencies are negligible, and

$$J \approx 10^{-4}, \quad \omega_{\theta} \approx 600, \quad \omega_x \approx 200.$$

These correspond to frequencies of  $100\text{Hz}$  and  $35\text{Hz}$  respectively, for rocking and for purely vertical motion.

## 6.2. The sound of a loadcell

We tapped the plate attached to a loadcell, and recorded the sound the system made as a result. The waveform was found to have a significant frequency component of about  $120\text{Hz}$ . This could resonate with the rocking frequency of the carriers, and thus explain why the higher frequency in the spectra of Figs (7) & (8) is always about  $120\text{Hz}$ .

## 7. Carrier Moment of Inertia

The moment of inertia of the floating part of a carrier was approximated by taking it apart and estimating the weight distribution very roughly. We found that  $J \approx 4 \times 10^{-5}\text{kg m}^{-2}$ . This corresponds to a rocking frequency of  $120\text{Hz}$  provided that (ignoring damping and using equation (4) ) the effective spring constant for a loadcell is  $k \approx 5000\text{kg/s}$ , which is comparable to the value obtained in section (6.1).

## 8. Conclusions and recommendations

We have studied the frequency components present in the output of loadcells, for various sized fruit running at various speeds. Apart from a high frequency which is of no concern to Compac, we typically see one or two lower frequencies, which reduce as fruit mass increases, causing difficulties with oscillations getting past the analogue filter.

We have developed models for simple harmonic motion in the vertical direction, and a side to side rocking motion between the two loadcells used to weigh the fruit. Our modelling suggests that a reduction in frequency is generally to be expected as mass increases.

One simple possibility for improving the estimation of fruit mass is to reduce the cutoff frequency of the lowpass filter. The one used by Compac for the data provided was set at a cutoff of about 60Hz. This could perhaps be reduced to 30Hz. However, this option might not help at higher operating speeds, as reducing the cutoff frequency means a slower response time for the filter, and there may not be enough time for the filtered signal to level off.

The key parameters are mass (and its distribution), effective spring constant, and effective damping. Other options are to stiffen and reduce the effective mass of the loadcells, thereby increasing oscillation frequency and damping. However, we understand that Compac have tried stiffer loadcells, which are rated for heavier masses. They then encounter difficulties associated with having to increase the amplification of the signal from the loadcell, and becoming more vulnerable to drift.

Compac could also consider stiffening and reducing the mass of the carriers themselves. The presence in data of an extra frequency in the lower range raises the question of whether flexing of the carriers might also be affecting the loadcell signals.

One promising strategy is to use the understandings we have gained from the modelling, rather than just filtering out the oscillations. We have shown that it is feasible in principle to infer key parameter values from the oscillation frequency, damping rate and oscillation amplitude. A joint approach, digitally combining this information with filtered output, might be faster and more accurate than the present setup.

Finally, a method that was considered during MISG'05, but which we did not have the expertise to develop further, is that described in [3, 2, 4], which uses an adaptive filtering technique. Such an approach apparently has a very fast response time, which may be useful for larger fruit weights and faster line speeds. It may be possible to develop an adaptive filter for the signal from a pair of load cells, using the model developed in §3. Using the mean signal from both cells will eliminate the higher-frequency rocking and simplify the adaptive filter required.

In any case, taking the average of both raw signals from the load cells before doing any processing is advisable. This will cancel the out-of-phase motion (rocking motion) and should give a cleaner signal with an oscillation frequency that depends on mass according to the usual simple harmonic motion (straight up-and-down).

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