Spinning Soccer Ball Trajectory

Problem Presenter
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Problem Statement

As can be remembered from the recent World Cup, uncertainities in the trajectory of a soccer ball at high speeds have led to some criticism on the ball manufacturers. The existing ball trajectory models assume that as the ball spins, a layer of air, say a boundary layer, follows the motion of the ball, thus spins with it. This, in turn, induces a velocity difference on the sides normal to ball's trajectory. The velocity difference then leads to pressure difference due to Bernoulli's principle. If $\vec{\omega}$ represents the axis of spin and $\vec{v}$ is the linear velocity, the resulting Magnus force would be in the direction of $\vec{\omega} \times \vec{v}$.

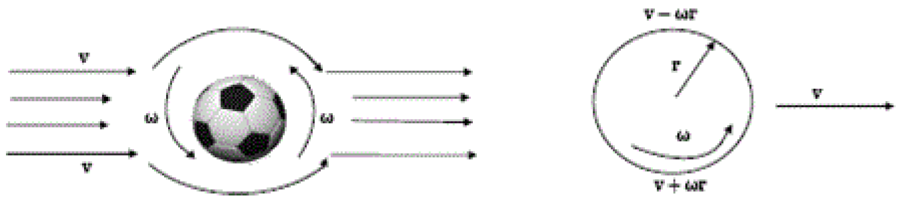

Figure. A soccer ball trajectory and the induced velocity difference

Yet, the resulting trajectory models do not seem to account for rapid changes in the trajectories. The study group is then asked to analyse the asumptions of the existing models and modify them if necessary to come up with a satisfactory model.

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Content

Abstract

In this study the trajectory of a soccer ball is investigated using a dynamical system that takes the Magnus, drag and gravitational forces into consideration. In particular, close attention is paid to the trajectory of a soccer ball under an initial rotational kick. We first note that balls which are rather smooth, such as the Jabulani, typically hit the critical conditions of the so-called "drag-crisis" at crutial moments in a game of soccer, such as during a free-kick, so that "knuckling" is more pronounced. We then propose a simplified system consisting of three ordinary differential equations describing horizontal and vertical acceleration and rotation rate as functions of the forces on the ball, and of its "roughness". We find that the parameter controlling the roughness to play a critical role in the resulting trajectory. In particular, when this parameter is small, as we assume it to be for soccer balls such as the Jabulani, it is possible for the trajectory to develop two turning points, suggesting that the ball could appear to "bounce" in mid-flight.

1. Statement of the problem

As can be remembered from the 2010 FIFA World Cup, uncertainties in the trajectory of the Jabulani soccer ball has resulted in some criticism of the ball's design. Existing models for the trajectory of spinning soccer balls assume that a layer of air, known as a boundary layer, follows the motion of the ball, and thus spins with it. This, in turn, induces a velocity difference on the sides normal to the ball's trajectory. The velocity difference results in a pressure difference due to Bernoulli's principle. If ω represents the axis of spin and v denotes the (linear) velocity, then the resulting Magnus force would be in the direction of $\omega \times v$. However, the resulting models do not sufficiently account for rapid changes in the trajectories. The study group is thus asked to analyse the assumptions of the existing models and assess their validity.

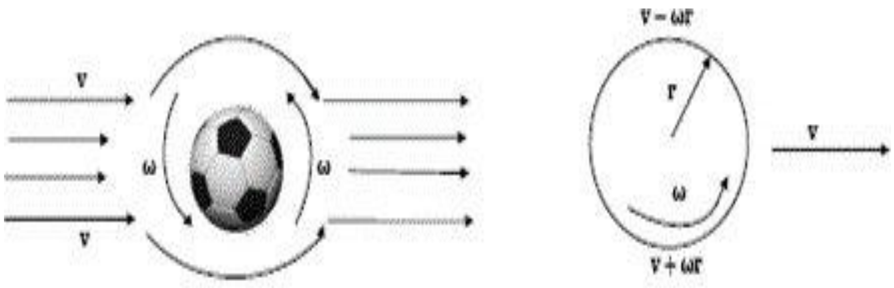

Figure 1: Velocity of a soccer ball.

2. Introduction and background

When a traditional soccer ball starts its descent, it is expected, due to the effect of gravity, to fall to the pitch without any reversal of its vertical velocity. However, the Jabulani has been observed to behave somewhat differently under certain conditions - after a forceful kick, soccer players have witnessed the ball moving upwards again shortly after the beginning of its fall; in other words, the ball's trajectory could have more that one maxima - "it's like putting the brakes on, but putting them on unevenly". This could clearly affect a player's ability to control the flight of the ball. From a goalkeeper's viewpoint, an approaching ball may suddenly appear to change its direction in a way grossly unpredictable. We believe this kind of phenomenon, arguably a much more common (and unwanted) attribute of the Jabulani, is related to the amount of, and indeed pattern of, the surface roughness of the ball. This in turn will affect the position of the separation points of the boundary layers, which are a feature rotating flows. Some specifications of a traditional soccer ball and the Jabulani ball are compared in Table 1. Although this table appears to show that the Jabulani has improved, more advanced, features, some of those who have used this ball at a competitive or professional level have criticised its controllability and dynamics.

	Standard FIFA approved ball	Jabulani
Circumference (cm)	68.5-69.5	69.0±0.2
Weight (g)	420-455	440± 0.2
Change in diameter (%)	≤1.5	≤1
Water absorbtion (weight increase, %)	≤10	1
Rebound test (cm)	≤10	≤6
Pressure loss (%)	≤20	≤10

Table1: Technical specification of standard FIFA approved soccer balls and the Jabulani ball.

The dynamics of a soccer ball in flight is closely related to a classical problem in theoretical fluids mechanics, namely the flow around a rotating sphere. For clarity, let us first consider a smooth cylinder of radius a in a stream with velocity U of ideal fluid with circulation Γ. The streamfunction in polar coordinates (r,θ) for this flow can be found to be [1]

$$\psi = Ur\sin\theta - \frac{Ua^2\sin\theta}{r} - \frac{\Gamma}{2\pi}\ln\frac{r}{a}, \qquad (1)$$

If $\Gamma \leq 4\pi Ua$, there is a stagnation point on the cylinder and from Bernoulli's principle it can be found that the drag F_D and lift F_L forces are respectively given by

$$F_D = 0, \tag{2}$$

$$F_L = -\rho U \Gamma, \tag{3}$$

where ρ is the density of the fluid. The lift force can be understood as follows: The circulation gives higher speed on one side of the cylinder (c.f Figure 1). This higher speed is associated with lower pressure p since

$$p + \frac{1}{2}\rho v^2 = \text{constant}, \tag{4}$$

and hence there is a force, known sometimes as the Magnus force, from the high pressure (slow speed) side to the low pressure (fast speed) side. So a soccer ball kicked with sufficient spin will generate lift and rise. As the ball slows its trajectory will be altered by the forces acting on it and curls upon descent, as is observed in, for example, free-kick taking. However, there are considerable viscous effects near the boundary of the cylinder/sphere and hence Bernoulli's principle loses its validity. The body may be subjected to turbulence, which can affect it's flight. Note that although this alters the forces on the sphere, the physical intuition remains. In this more complicated, yet more interesting and realistic case, the forces acting on the body are given by

$$F_D = -\frac{1}{2} C_d \rho A |v|^2 e_x, \tag{5}$$

$$F_M = -2 C_d \rho A |v||\omega| e_y, \tag{6}$$

$$F_g = Mg, \tag{7}$$

where C_d is the drag coefficient, ρ is the fluid (air) density, A is the cross-sectional area of the ball, M is its mass, and $g = 9.8\,\mathrm{m/s^2}$ is the gravity. The drag coefficient C_d will depend on the properties of the ball and the Reynolds number, Re, which is defined to be

$$Re = \frac{|v|L}{v}, \tag{8}$$

where v is the fluid's kinematic viscosity and L is a characteristic length, which is taken to be the diameter of the ball. Moreover, at high Reynolds numbers, boundary layers are present and flow separation is observed at two separation points on the cylinder. In the absence of rotation these separation points are symmetric with respect to the azimuthal coordinate (see Figure 2, for example) and asymmetric when the cylinder/sphere is spinning.

The displacement of the line of separation has a considerable effect on the flow. In Figure 2 the separation points are close together so that the turbulent wake beyond the body is contracted. This, in turn, reduces the drag experienced by the body. Thus the onset of turbulence in the boundary layer at larger Reynolds numbers is accompanied by a decrease in the drag coefficient. When the separation points are further apart the drag increases significantly (this is sometimes referred to as the drag crisis [2]). In other words, causing a turbulent boundary layer to form on the front surface significantly reduces the sphere's drag. In terms of soccer balls, for a given diameter and velocity the manufacturer has just one option to encourage this transition: to make the surface rough in order to create turbulence. We note that the same principal applies to golf balls.

Although the mathematics and physics of rotating bodies is complicated, we develop a simple dynamical system to describe the trajectory of such a body that is affected by acceleration, spin, and surface roughness. Despite its simplicity it can highlight the motion of a soccer ball and the critical features that can cause unexpected or unwanted behaviour. We point the interested reader to other works in this field [4, 5, 6, 7].

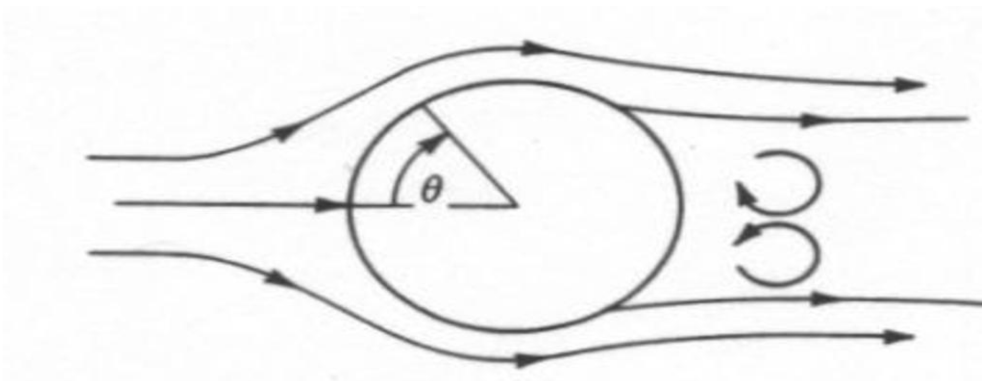

Figure 2: The general character of flow over a cylinder at high Reynolds numbers.

The remained of this report is structured as follows: In Section 3 we note some observations we have made related to the flight of the Jabulani; in Section 4 we present the dynamical system for the trajectory of a rotating cylinder; the results are presented in Section 5 and a summary is drawn in Section 6.

3. Observations

The relationship between the drag coefficient and Reynolds number for smooth and rough spherical bodies is shown in Figure 3.

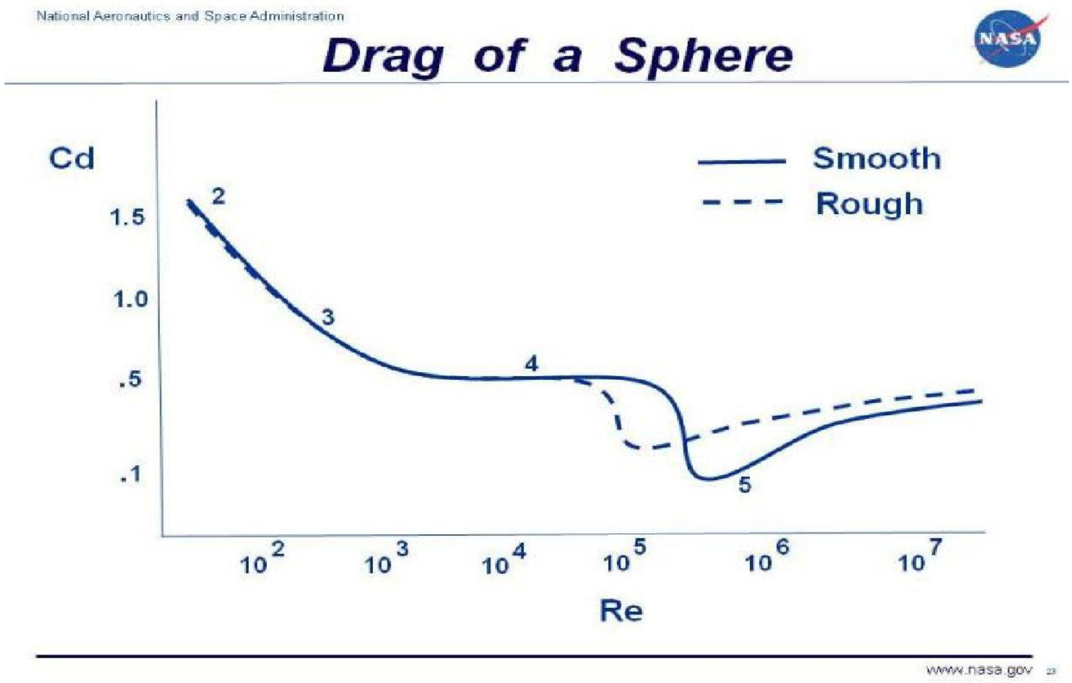

Figure 3: Drag coefficient for rough and smooth spheres.

Traditional soccer balls are not smooth. Although some surface roughness has been added to the Jabulani it is still a lot smoother than other soccer balls. It is therefore reasonable to assume that the dynamics of each will differ, especially when rotation is included. Also, traditional balls have a hexagonal design and at small spin the stitching can cause an a asymmetric flow field, causing the ball to "knuckle"; that is it may be pushed a small amount in a given direction even when the ball has little rotation. The Jabulani has no stitching and the speed at which knuckling may occur is $20 - 30\,\mathrm{km/h}$ faster with the Jabulani than with traditional balls [3]. This coincides with the average speed of a free kick, and hence the pronounced visibility of chaotic trajectories with the Jabulani.

During its flight, the highest measured speed of a soccer ball kicked by a player in an official game is $140\,\mathrm{km/h}$. A typical powerful shot kicked by a professional player (during a free-kick, for example) gives the ball a speed of about $100\,\mathrm{km/h}$ (about $30\,\mathrm{m/s}$). With the known values of v (roughly $20 \times 10^{-6}\,\mathrm{m^2/s}$) and L, we calculate the Reynolds number in to be between between $100\,000$ and $500\,000$. From Figure 3 we can see that the drag coefficient of the rough ball does not drop as dramatically as the smooth one in this regime. Although the Jabulani does have some surface roughness, it is still considerably smoother than any other soccer ball. Therefore the Jabulani may experience rapid changes in the forces acting on it during high Reynolds number flows, such as during a free kick.

We also note that the distribution of surface roughness is likely to effect the flow field around a rotating body. Old soccer balls may have stitching which could potentially alter the flow field, but the stitching is evenly spread over the surface area. It is not obvious if the small roughness that has been added to the Jabulani is equally distributed over the ball. Indeed, an eye-ball examination would suggest otherwise. Although this is unlikely to have an effect in most situations it may well be an important factor that needs consideration when the ball is rotating at high Reynolds numbers.

4. The proposed model

We developed a simple, idealised, dynamical system for the trajectory of a cylindrical body that takes surface roughness into consideration. The governing system is written as

$$\ddot{x} = -\frac{1}{2m} C_d \rho A \dot{x} \sqrt{\dot{x}^2 + \dot{y}^2} - \frac{2\rho A r}{m} C_d \dot{y} \omega \,, \tag{9}$$

$$\ddot{y} = -\frac{1}{2m} C_d \rho A \dot{y} \sqrt{\dot{x}^2 + \dot{y}^2} + \frac{2\rho A r}{m} C_d \dot{x} \omega - g, \tag{10}$$

$$\dot{\omega} = -\frac{R}{r} \sqrt{\dot{x}^2 + \dot{y}^2}\,\omega, \tag{11}$$

subject to the following initial conditions:

$$x(0) = 0; \; x'(0) = 30, \tag{12}$$

$$y(0) = 2; \; y'(0) = 0, \tag{13}$$

The initial conditions for ω will be discussed in the following section. In the above, R is a parameter (assumed constant) describing the roughness of the ball, r is the radius and x, y and ω are functions of time (t). We assume the drag coefficient undergoes a rapid change in the critical Reynolds number regime and fit it using a cubic interpolation from Figure 3, that is

$$C_d = 0.0198 \left(\frac{Re^3}{3} - \frac{7Re^2}{2} + 6Re \right) + 0.457, \tag{14}$$

The other parameters were chosen according to the physical characteristics:

$$m = 0.44 \text{ kg},$$

$$r = 0.1098 \text{ m},$$

$$\rho = 1.225 \text{ kgm}^{-3},$$

$$A = 0.037875167 \text{ m}^2,$$

$$v = 2 \times 10^{-5} \text{ m}^2\text{s}^{-1}$$

The first two equations in our system my be viewed as statements of Newton's second law of motion, i.e force = mass×acceleration, where the horizontal and vertical forces are taken from equations 5 and 6. Equation (9) describes the horizontal acceleration in terms of the drag force. We see that the equation for vertical motion, equation (10), has three terms; the first two of these (which correspond to the Magnus, or lift, force) must exceed the last (which is describing the action of gravity) if there is to be an upwards motion. It should also be noted that the second terms in equations (9) and (10) depend on the rotation, ω, which itself depends on the roughness parameter R.

5. Results: Trajectory of the ball

We consider the trajectory of smooth and rough balls with different initial conditions. That is, we solve the dynamical system given my equations (9)-(11) subject to $\omega(0)$. When R is relatively large, or when the ball is "rough", the gravity term dominates the solution for the vertical path and, once the ball has reached its maximum height (here there is only one local maximum of the trajectory), it starts to uniformly descend, as shown by the dash-dotted line in Figure 4. This is what one would expect if playing soccer with a sensible ball in sensible conditions. Similar results are obtained even when the ball is given a relatively hefty rotational kick ($\omega(0) \sim 50$) but will behave curiously for excessively (unrealistic) large initial vales of ω. If we reduce R, which corresponds to a smoother ball, a sufficiently (but not excessively) large amount of initial spin can cause the body to generate a secondary lift (Magnus force) as it begins its descent, which actually causes it to rise again briefly – a phenomena that has been observed with the Jabulani and predicted by Figure 4 (solid line) and Figure 5 using our model. The dotted line in Figure 4 is the predicted trajectory when $R = 0.00002$ and $\omega(0) = 50$ – that is, a very smooth ball with large initial rotation. In this case we see a steep rise to a global maximum of the trajectory, which is beyond (and higher than) the local maximum of a standard parabolic curve under comparable conditions. This is an extreme case and may not be realisable in practise.

For further insight we plot the velocity and acceleration for different values of R when $\omega(0) = 50$ in Figures 6 and 7, respectivley. The increase in the velocity and decrese in acceleration when R is small is noteworthy since it may be

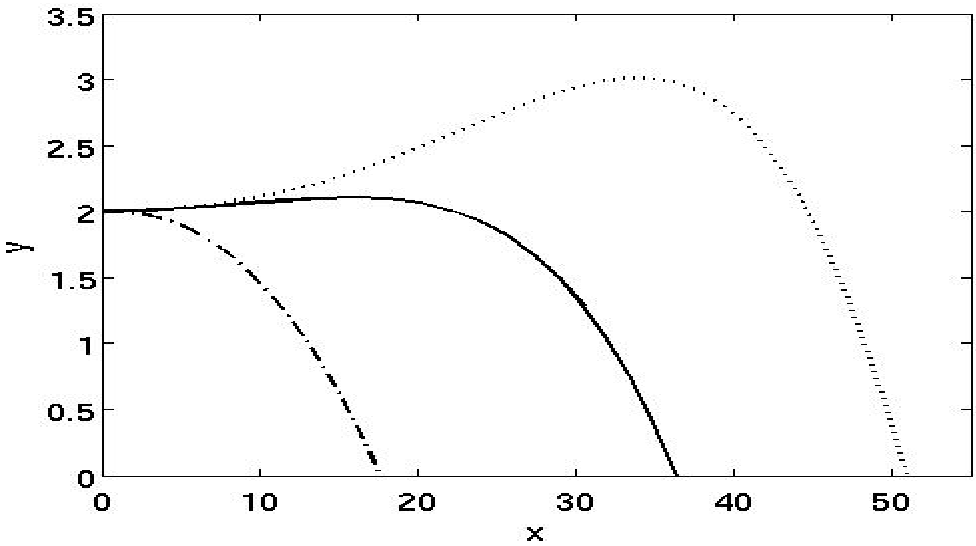

Figure 4: The $x-y$ trajectory of a spinning ball when $R=0.00002$ (dotted), $R=0.002$ (solid), $R=0.2$ (dash-dotted) with initial angular velocity $\omega=50$.

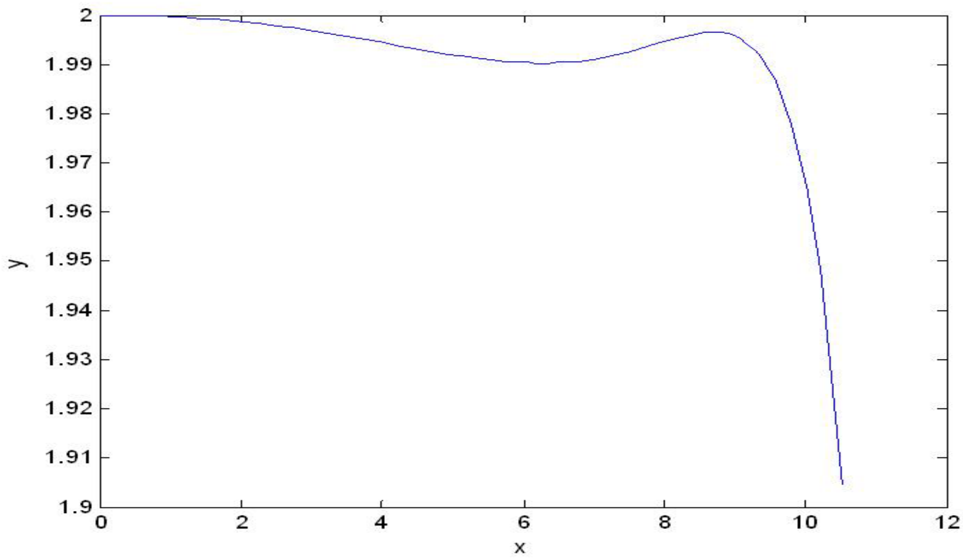

Figure 5: The $x-y$ trajectory of a spinning ball obtained from our dynamical system.

counter-intuitive and certainly not what one expects with traditional, rougher, soccer balls (c.f dot-dashed lines in Figures 6 and 7).

We mentioned above that the initial conditions (in partcular, the initial rotation) also affect the flight path, and this can be seen in Figures 8, 9 and 10. Figure 10 is particuarly enlightening since it shows the complicated, highly non-linear behaviour of the acceleration for a smooth ball, even when the initial rotation is relatively small.

6. Discussion

We have proposed a dynamical system to predict the trajectory of a cylindrical body subject to acceleration, spin, and surface roughness. We have shown that the roughness (included through the parameter R) and the initial spin to be the critical factors responsible for the so-called "knuckling" effect, and for unpredictable changes in a balls vertical acceleration. This could offer some understanding to why the Jabulani, a smoother-than-normal ball, exhibits somewhat unpredictable behaviour under crucial conditions such as free-kick taking, long-range passing, and distance shooting. Physically, the roughness parameter R is related to not only the surface structure of the ball but also the separation points of the boundary layer during rotational flow. Small R will be accompanied by widely spaced separation points and a large turbulent wake, whereas a larger R means the separation points will be closer and the wake behind the sphere narrower. A potentially interesting further study could be to try and quantify this relationship, perhaps by introducing a fourth equation for the symmetry of separation or to include the "spread" of roughness (for example, as noted above it is not clear if the roughness purposefully added to the Jabulani is sufficiently, or evenly, distributed over the ball's surface). Perhaps a model with a variable R may also be enlightening. We have also remarked that the forces on the Jabulani may undergo rapid changes (more rapid than for rougher traditional balls) during a free kick, which will effect its flight kinematics. It is also worth mentioning that an experimental study [3] revealed that the new ball falls victim to "knuckling" at higher velocities than old soccer balls, which coincides with the average maximum speed of flight during a free kick. It is unclear if the knuckling effects are stronger with the Jabulani, or just more readily observed. Considering free kicks and high

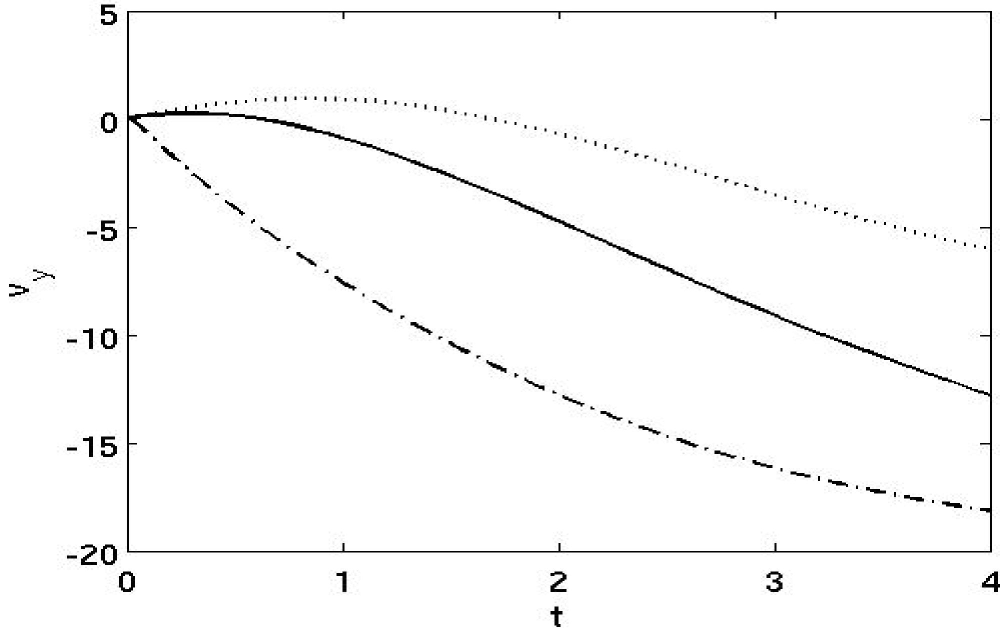

Figure 6: The vertical velocity of a spinning ball when $R = 0.00002$ (dotted), $R = 0.002$ (solid), $R = 0.2$ (dash-dotted) with initial angular velocity $\omega = 50$.

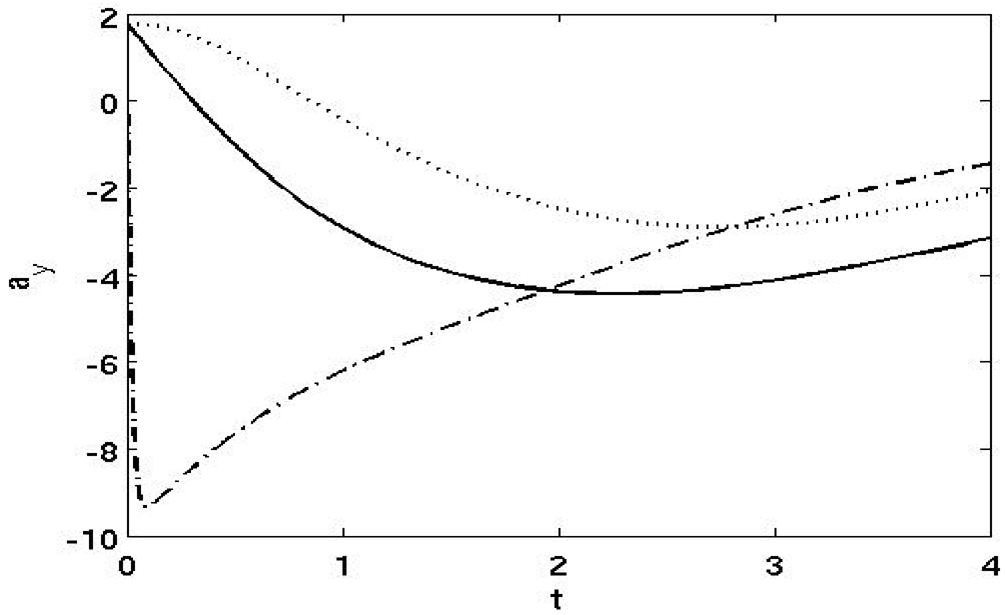

Figure 7: The vertical acceleration of a spinning ball when $R = 0.00002$ (dotted), $R = 0.002$ (solid), $R = 0.2$ (dash-dotted) with initial angular velocity $\omega = 50$.

velocity passing and shooting are crucial elements of soccer, one would desire to have maximum control during these critical moments.

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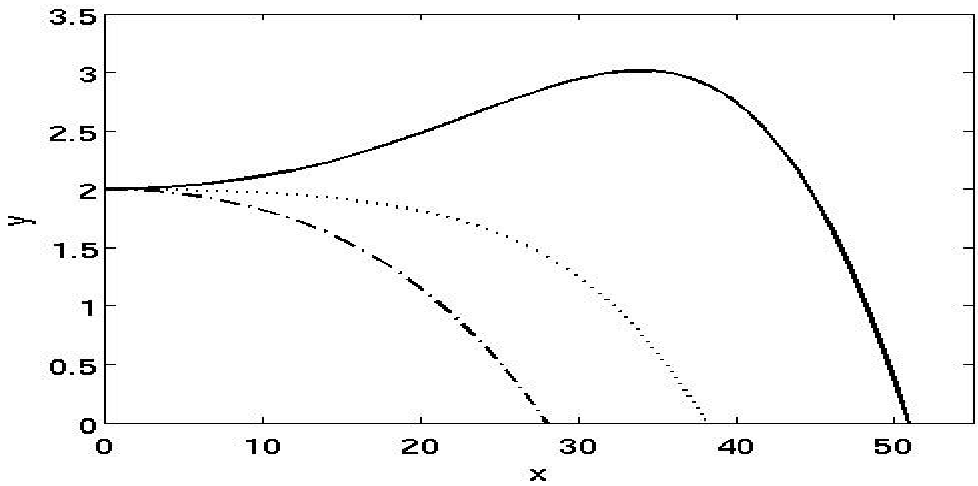

Figure 8: The $x - y$ trajectory of a spinning ball when $\omega = 50$ (solid), $\omega = 40$ (dotted), $\omega = 30$ (dash-dotted) with roughness parameter $R = 0.00002$.

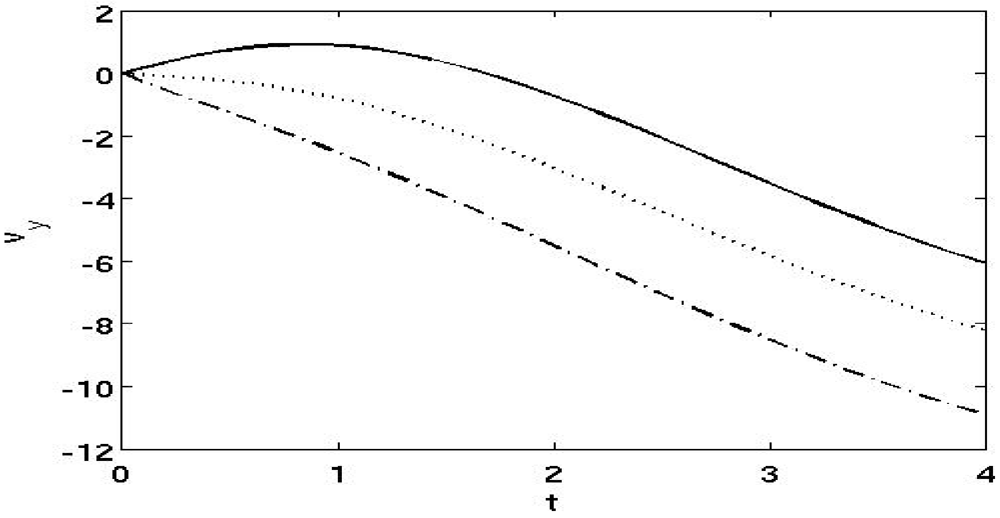

Figure 9: The vertical velocity of a spinning ball when $\omega = 50$ (solid), $\omega = 40$ (dotted), $\omega = 30$ (dash-dotted) with roughness parameter $R = 0.00002$.

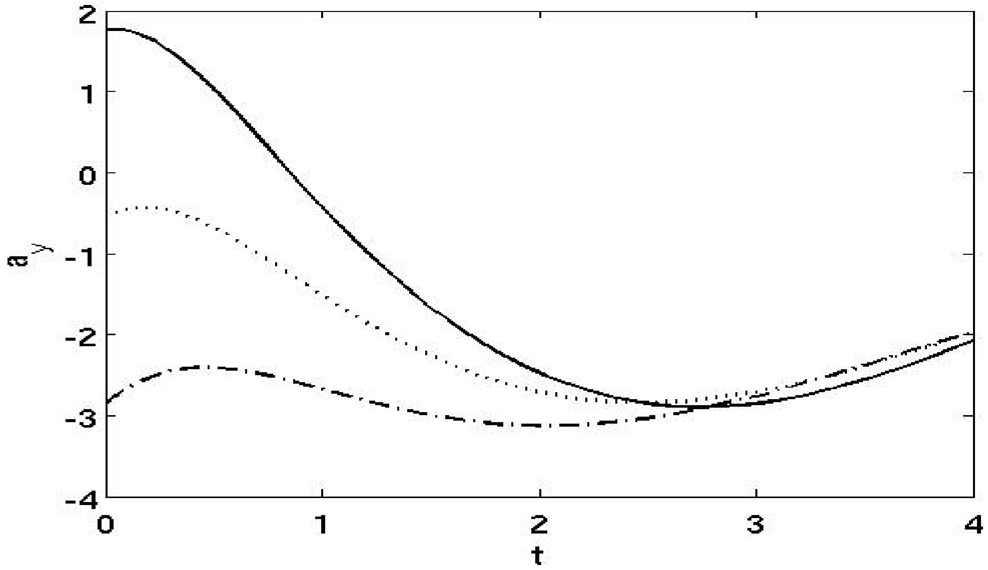

Figure 10: The vertical acceleration of a spinning ball when $\omega = 50$ (solid), $\omega = 40$ (dotted), $\omega = 30$ (dash-dotted) with roughness parameter $R = 0.00002$.