

# Minimum Particle Size for Cyclone Dust Separator

Perkins Technology

## 1 Background

Perkins technology wish to separate small soot particles from exhaust gases, and the question posed to the study group was to determine the feasibility of using a cyclone separator to remove these particles. Soot is mostly composed of polycyclicaromatic compounds and results from the incomplete combustion of the diesel fuel in the engine. The average size of the particles formed in the engine is in the range 3 to 10 nm in diameter, but this is known to increase within the exhaust system.

In the first part of this report we determine the minimum particle size that can be removed by centrifugal separation. The second part discusses the mechanisms for particle growth within the exhaust system in order to estimate the particle growth rate.

## 2 The effect of particle size on cyclone separation

The principal behind the cyclone separator is to use centrifugal force to separate phases of different densities. The fluid in the cyclone is forced to rotate rapidly so that the centrifugal acceleration causes the denser particles to migrate towards the outside of the cyclone (see figure 1), where they are collected. The condition for effective separation is, therefore, that this migration speed is sufficiently rapid to separate the particles within the residence time of the fluid in the cyclone.

The centrifugal force acting on a spherical particle of diameter  $d$  is given by

$$\frac{\pi}{6}(\rho - \rho_{\text{air}})d^3r\omega^2$$

where  $\rho$  and  $\rho_{\text{air}}$  are the densities of the particles and air respectively,  $\omega$  is the angular velocity of the air in the cyclone and  $r$  is radial distance from the centre of the cyclone. As the particle density is much larger than that of the air, we may neglect  $\rho_{\text{air}}$  from this expression.

For small particles, the effect of fluid inertia on the lengthscale of the particle may be neglected, so that the drag force on the particle is  $3\pi\mu du$  where  $\mu$  and  $u$  are the viscosity and particle velocity relative to the fluid respectively. Therefore, the radial drift velocity,  $u_d$ , of particles with respect to the fluid satisfies

$$\frac{\rho\pi d^3}{6}\dot{u}_d + 3\pi\mu du_d = \frac{\rho\pi d^3}{6}r\omega^2. \quad (1)$$

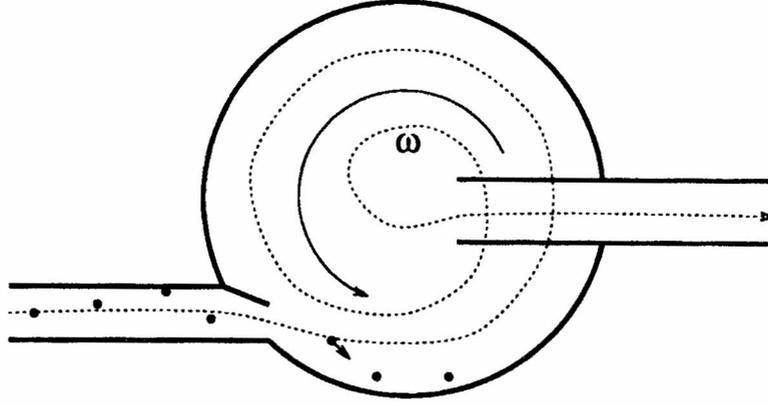


Figure 1: Sketch of the cyclone separator.

where dot denotes differentiation with respect to time. Thus the maximum drift velocity  $u_d$  of a particle is given by

$$u_d = \frac{\rho d^2}{18\mu} R\omega^2 \quad (2)$$

where  $R$  is the radius of the cyclone.

If the particles are to be removed from the airstream, their drift velocity must be sufficient for them to reach the outer part of the cyclone before the air exits the cyclone. If the airstream turns through an average angle of  $\alpha$  radians within the cyclone, the residence time in the cyclone will be  $\alpha/\omega$ . The precise value of  $\alpha$  depends upon the details of the flow within the cyclone, and important aspect of cyclone's design should be to make  $\alpha$  as large as possible.

In this time the particles must move radially by the width of the airstream. As this distance will scale with the size of the cyclone it is convenient to write this as  $\beta R$ . Again  $\beta$  will depend upon the design detail of the cyclone. The condition for particles to be removed is therefore

$$u_d \geq \frac{\beta}{\alpha} R\omega. \quad (3)$$

and substituting from equation (2) we require that

$$\frac{\rho\omega d^2}{\mu} \geq 18\frac{\beta}{\alpha}. \quad (4)$$

From this condition we can estimate the minimum angular velocity required to separate particles of a given size. The particle density and air viscosity are approximately  $2 \times 10^3 \text{kg m}^{-3}$  and  $2 \times 10^{-5} \text{Pa s}$  respectively, and taking  $18\beta/\alpha$  to be unity, we obtain the condition that

$$\omega \geq \frac{10^{-8}}{d^2} \text{ m}^2\text{s}^{-1}$$

Thus, for one micron diameter particles the angular velocity must exceed  $10^4$  radians per second (*i.e.*  $10^5$  r.p.m.), while 0.1 micron diameter particles require  $10^6$  radians

per second. While it is possible that careful design could reduce the value of  $\beta/\alpha$  below 0.05, it is unlikely that more than a factor of 10 improvement can be obtained. The figure of one hundred thousand r.p.m. is likely to be the upper limit that can be produced in a device of this kind, giving a minimum particle size of around one micron.

It should be noted that this estimate is based on spherical particles. If the particles are elongated or have an open fractal structure, the drift velocity will be smaller as it is proportional to the ratio of the particle mass to its greatest linear dimension.

Small particles are also subject to Brownian motion which gives rise to random particle motion with an average mean square velocity of magnitude

$$\sqrt{\frac{6kT}{\pi\rho d^3}}$$

(where  $k$  is the Boltzmann constant and  $T$  is the absolute temperature) and so a further condition on  $u_d$  is that

$$u_d^2 > \frac{6kT}{\pi\rho d^3}.$$

However, this value of  $u_d$  is much smaller than the minimum drift speed obtained from equation (3) above.

### 3 Growth in particle size

As they enter the exhaust system the soot particles have diameters in the range 3 to 10 nm, and so are much smaller than the minimum size of about 1 micron that can be removed by the cyclone. However, it is known that mean particle size increases within the exhaust system from a combination of the condensation of gaseous hydrocarbons onto the surface of the soot particles and aggregation of soot particles.

#### 3.1 Condensation

The surface of the soot particles readily accepts other hydrocarbons and so the soot particles act as nucleation sites for the condensation of gaseous hydrocarbons as the exhaust gases cool. This effect of increasing soot mass is opposed by surface oxidation of the soot. The members of the study group did not feel qualified to discuss these issues, but observations of soot particles in the literature (*e.g.* H.F. Calcote, (1981) Mechanisms of soot nucleation in flames — a critical review, *Combustion and Flame* **42** pp 215–242) find that the individual spherical soot particles grow to between 10 to 50 nm and then aggregate to form larger compound soot particles. Condensation is probably the explanation for the observation made by Perkins Technology that the particle size increases if the exhaust gases are cooled.

#### 3.2 Aggregation

The primary method of particle growth is by aggregation when two soot particles collide and fuse. The growth-rate is therefore determined by the rate at which

particles collide with one another as a result of Brownian motion.

The simplest theory (due to Smoluchowski) calculates the collision-rate  $J$  between spherical particles of diameter  $d$  and number density  $n$  from the probability of one particle diffusing to within a distance  $d$  of another, and gives

$$J = 4\pi d D_{\text{pair}} n^2 \quad (5)$$

where  $D_{\text{pair}}$  is the pair diffusivity equal to twice the self-diffusivity for a particle of diameter  $d$ . Hence for a dilute suspension  $D_{\text{pair}}$  is given by  $2kT/3\pi\mu d$ , ignoring hydrodynamic interactions between particles. If we assume further that all colliding particles fuse together, then the aggregation rate is given by

$$-\frac{dn}{dt} = \frac{8kT}{3\mu} n^2$$

Note that this is independent of particle diameter. Although smaller particles diffuse more rapidly, their small size means that the particles must come closer to one another in order to aggregate. If we express the aggregation in terms of the mean particle volume  $v$ , where  $nv = \phi$  is the volume concentration of soot, we obtain

$$\frac{dv}{dt} = \frac{8kT}{3\mu} \phi, \quad (6)$$

so that for a fixed volume concentration of soot,  $\phi$ , the aggregation rate is constant. Thus, if the initial particle volume is small, the mean particle diameter after a time  $t$  is given by

$$d = \frac{6}{\pi} \left( \frac{8kT\phi t}{3\mu} \right)^{1/3}. \quad (7)$$

At a temperature of 1000K and a volume concentration of soot of 0.1 ppm we obtain an estimate for  $\frac{dv}{dt}$  of approximately  $2 \times 10^{-22} \text{m}^3 \text{s}^{-1}$ . Thus in one second the average particle diameter will grow to approximately 70 nm.

If the particle size distribution is polydisperse the aggregation rate will be larger. In the Smoluchowski equation the centre-to-centre distance at which the particles collide is the average of their diameters and the pair diffusivity is the sum of the self-diffusivities of the two particles. Thus the collision rate increases by a factor of  $(d_1 + d_2)^2/4d_1d_2$ , where  $d_1$  and  $d_2$  are the particle diameters.

A question arises as to whether the continuum approximation is valid for such small particles. The mean-free path in air is approximately 100nm and so the Knudsen based on the original diameter of the soot particles is greater than 10. However, the aggregation is controlled by the mean distance between particles, which is of the order of a micron and is therefore large compared to the mean-free path. In any case the alternative calculation based on free molecular flow (see B.S. Haynes and H.G. Wagner (1981) Soot formation, *Prog. Energy and Combustion Sci*, 7, 229–273) gives similar results for the particle growth rate with an average particle diameter of 100nm after 1 second.

The measurements of particle size by Perkins technology at the end of a long exhaust system find mean particle diameters in the range 60 to 100 nm, in line with the theoretical estimate from equation (7). From this equation it can be seen

that the diameter of the particles increases weakly with gas temperature, particle volume fraction and time, so that the particle size could be increased by increasing the length of the exhaust system or the particle volume concentration. However, the product of these parameters would have to increase by a thousand-fold to produce a one micron particle.

A further possible mechanism for aggregation is ballistic aggregation by larger particles within the cyclone. As these large particles migrate towards the outside of the cyclone they will collide with smaller particles that are moving with the air-stream. The volume of air swept by a migrating particle of diameter  $d$  will be  $\pi d^2 \beta R / 4$  and so, on the assumption that all the small particles in this volume aggregate with the large particle, the fractional increase in mass of the larger particle will be

$$\frac{\Delta m}{m} = \frac{3\phi\beta R}{2d}.$$

As  $d$  must be at least 1 micron and  $\phi = 10^{-7}$ , the fractional increase in mass, for a cyclone with  $\beta R$  equal to 6 cm or less, will be less than one percent.

If the soot particles carry a net charge, the aggregation rate will be reduced due to electrostatic repulsion. Haynes and Wagner cite experiments in which aggregation of the soot particles is suppressed by adding caesium or potassium salts. Aggregation will be enhanced if the soot particles carry an electric dipole, and will tend to aggregate into long chains, as is observed in some experiments. This suggests an alternative means of separating the soot using an electric field.

## 4 Summary

In section two we estimated the minimum particle diameter that can be removed by a cyclone separator is around one micron. This estimate is consistent with current applications of hydrocyclones. The particle size measurements by Perkins Technology together with our estimates from section three, suggest that the soot particles are an order of magnitude smaller than this. Although it may be possible to remove some particles less than one micron in diameter with a well designed high-speed cyclone, we do not think it will be possible to remove a substantial proportion of 100nm or smaller particles.

The growth rate of the particles increases if the particles volume fraction or the polydispersity is increased. Therefore aggregation could be enhanced by the addition of larger particles ( $d > 1\mu\text{m}$ ) or water droplets (provided the water doesn't all vapourise) to the exhaust gas.

## List of Participants

The study group participants who worked on the two problems presented by Perkins Technology included, John Byatt-Smith, Richard Day, Oliver Harlen, Sam Howison, John Lister and Stefan Llewellyn Smith. We would also like to thank Richard Stone for sharing his expert knowledge of diesel engines with us.

