Study Group Report
Conductor Mounted Residual Current Detector

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1 Introduction

Overhead power transmission lines consist of 3 cables carrying 3-phase current. The sum of the currents in the three cables, known as the residual current, should therefore be zero at all times. A non-zero residual current is a good indication that there is a fault somewhere down the line (for example, one of the cables may have earthed).

The problem brought to the study group concerned the measurement of the residual current by means of a detector mounted on the centre cable. The detector would comprise a number of probes, each of which could sample the magnetic field (in a particular direction) due to the current flowing in the cables, mounted on arms attached to a torroidal search coil which would be mounted around the centre cable (see fig. 1). The torroidal search coil would be able to detect accurately the magnetic field due to the current flowing in the central wire, and hence this current, which we shall denote by I_3, can be taken to be known. The readings from the probes would then be used to calculate the remaining currents I_1 and I_2.

The problem to be addressed was of the effect of the wind. The detector would be mounted near to a support pole, so that the wires themselves would not move a significant amount in the wind. However, the detector itself may blow in the wind, resulting in a rotation of the whole apparatus about the centre wire. This would then lead to an erroneous measurement, and the detector would detect a fault when there was none. Thus the problem was to find a configuration of probes, and an algorithm for calculating the residual current from their measurements, which is robust with respect to rotations of the whole configuration about the centre wire.

2 Formulation

Figure 2 shows the three cables and a general configuration of four probes attached to the centre cable, which has rotated through an angle θ from it's rest position. Each probe has a particular orientation, and measures the magnetic flux threading it (with the arrow indicating the positive direction). Each wire generates a magnetic field

$$\frac{I_i}{r}e_\phi,$$

where I_i is the current down the wire, r, ϕ are polar coordinates centred on the wire, and e_ϕ is the unit vector in the azimuthal direction. If we denote by B_i the reading of the ith probe, we therefore have

$$\begin{aligned}
B_1 &= \frac{\cos(\alpha_1 - \beta_1 - \gamma_{11} + \theta)}{x_{11}}I_1 - \frac{\cos(\alpha_1 - \beta_1 + \gamma_{21} + \theta)}{x_{21}}I_2 + \frac{\cos\beta_1}{r_1}I_3, && (1)\\
B_2 &= \frac{\cos(\alpha_2 - \beta_2 + \gamma_{12} - \theta)}{x_{12}}I_1 - \frac{\cos(\alpha_2 - \beta_2 - \gamma_{22} - \theta)}{x_{22}}I_2 - \frac{\cos\beta_2}{r_2}I_3, && (2)\\
B_3 &= \frac{\cos(\alpha_3 - \beta_3 - \gamma_{13} - \theta)}{x_{13}}I_1 - \frac{\cos(\alpha_3 - \beta_3 + \gamma_{23} - \theta)}{x_{23}}I_2 + \frac{\cos\beta_3}{r_3}I_3, && (3)\\
B_4 &= \frac{\cos(\alpha_4 - \beta_4 + \gamma_{14} + \theta)}{x_{14}}I_1 - \frac{\cos(\alpha_4 - \beta_4 - \gamma_{24} + \theta)}{x_{24}}I_2 - \frac{\cos\beta_4}{r_4}I_3, && (4)
\end{aligned}$$

$$\begin{aligned}
\sin\gamma_{ij} &= \frac{r_j}{x_{ij}}\sin(\alpha_j \pm \theta), && (5)\\
\cos\gamma_{ij} &= \frac{d + (-1)^{i+j}r_j\cos(\alpha_j \pm \theta)}{x_{ij}}, && (6)\\
x_{ij} &= \sqrt{d^2 + r_j^2 + 2(-1)^{i+j}dr_j\cos(\alpha_j \pm \theta)}, && (7)
\end{aligned}$$

where the plus sign is taken for $j = 1, 4$, and the minus sign is taken for $j = 2, 3$. In principle we have only three unknowns, and can therefore solve the system completely with only three probes in any configuration. However, in practice, it may not be so easy to solve for the angle θ. The problem is then to find a configuration of probes in which the angle θ can be easily approximated.

Things are much simplified if we consider four probes, in two pairs, such that the probes in each pair are equal and opposite. This corresponds to choosing

$$\alpha_4 = \alpha_1, \qquad \alpha_3 = \alpha_2, \qquad \beta_4 = \beta_1, \qquad \beta_3 = \beta_2,$$

$$r_4 = r_1, \qquad r_3 = r_2,$$

and implies that

$$x_{14} = x_{21}, \qquad x_{24} = x_{11}, \qquad x_{13} = x_{22}, \qquad x_{23} = x_{12},$$

$$\gamma_{14} = \gamma_{21}, \qquad \gamma_{24} = \gamma_{11}, \qquad \gamma_{13} = \gamma_{22}, \qquad \gamma_{23} = \gamma_{12},$$

Then, subtracting equation (4) from equation (1), and equation (3) from equation (2) we find

$$\begin{aligned} B_1 - B_4 &= (I_1 + I_2)\left(\frac{\cos(\alpha_1 - \beta_1 - \gamma_{11} + \theta)}{x_{11}} - \frac{\cos(\alpha_1 - \beta_1 + \gamma_{21} + \theta)}{x_{21}}\right) + \frac{2\cos\beta_1}{r_1} I_3, \quad (8) \\ B_2 - B_3 &= (I_1 + I_2)\left(\frac{\cos(\alpha_2 - \beta_2 + \gamma_{12} - \theta)}{x_{12}} - \frac{\cos(\alpha_2 - \beta_2 - \gamma_{22} - \theta)}{x_{22}}\right) - \frac{2\cos\beta_2}{r_2} I_3. \quad (9) \end{aligned}$$

We can eliminate $(I_1 + I_2)$ to obtain the following equation for θ:

$$\begin{aligned} &\left(\frac{\cos(\alpha_1 - \beta_1 - \gamma_{11} + \theta)}{x_{11}} - \frac{\cos(\alpha_1 - \beta_1 + \gamma_{21} + \theta)}{x_{21}}\right) = \\ &\quad \left(\frac{B_1 - B_4 - 2(\cos(\beta_1)/r_1)I_3}{B_2 - B_3 + 2(\cos(\beta_2)/r_2)I_3}\right)\left(\frac{\cos(\alpha_2 - \beta_2 + \gamma_{12} - \theta)}{x_{12}} - \frac{\cos(\alpha_2 - \beta_2 - \gamma_{22} - \theta)}{x_{22}}\right) (10) \end{aligned}$$

Here we see the simplification introduced by our choice of geometry: θ is dependent only on the ratio

$$\frac{B_1 - B_4 - 2(\cos(\beta_1)/r_1)I_3}{B_2 - B_3 + 2(\cos(\beta_2)/r_2)I_3},$$

which we shall henceforth call λ. In principle the procedure is now to solve (10) for θ as a function of λ, and substitute this into either (8) or (9) to give $(I_1 + I_2)$ as a function of B_1,

B_2, B_3, B_4 and I_3. In practice however, (10) is hard to solve in closed form. We also have the constraint that the resulting function $(I_1 + I_2)(B_1, B_2, B_3, B_4, I_3)$ must be such that it can be evaluated easily by the device.

We simplify our notation by setting

$$f_1(\theta) = \left(\frac{\cos(\alpha_1 - \beta_1 - \gamma_{11} + \theta)}{x_{11}} - \frac{\cos(\alpha_1 - \beta_1 + \gamma_{21} + \theta)}{x_{21}}\right), \tag{11}$$

$$f_2(\theta) = \left(\frac{\cos(\alpha_2 - \beta_2 + \gamma_{12} - \theta)}{x_{12}} - \frac{\cos(\alpha_2 - \beta_2 - \gamma_{22} - \theta)}{x_{22}}\right), \tag{12}$$

so that

$$B_1 - B_4 - \frac{2\cos\beta_1}{r_1} I_3 = (I_1 + I_2) f_1(\theta), \tag{13}$$

$$B_2 - B_3 + \frac{2\cos\beta_2}{r_2} I_3 = (I_1 + I_2) f_2(\theta), \tag{14}$$

and

$$\lambda = \frac{f_1(\theta)}{f_2(\theta)}. \tag{15}$$

Inverting this equation gives $\theta = \theta(\lambda)$. Substituting into (14) gives

$$\begin{aligned} I_1 + I_2 &= \left(B_2 - B_3 + \frac{2\cos\beta_2}{r_2} I_3\right) \frac{1}{f_2(\theta(\lambda))}, \\ &= \left(B_2 - B_3 + \frac{2\cos\beta_2}{r_2} I_3\right) g(\lambda), \end{aligned} \tag{16}$$

say. The problem is to find and approximate $g(\lambda)$. Note that if a linear approximation is used for $g(\lambda)$, then this will result in an approximation to $I_1 + I_2$ which is linear in B_1, B_2, B_3, B_4, and I_3. Notice also that the approximation of g is made distinctly easier by the fact that it is a function of one variable only. If g were a general function of $B_1, \cdots, B_4$, I_3 the calculations involved in making an approximation would be considerably more difficult.

The difficult part in the above procedure is inverting equation (15). However, approximating the solution of (15) directly is not the best way to proceed, since $1/f_2(\theta)$ is sufficiently complicated that it too will need to be approximated in order to be evaluated by the device. It is better to solve (15) exactly (maybe numerically), to give $g(\lambda)$ exactly, and then to approximate.

In the next section we illustrate the procedure with an example in which $g(\lambda)$ can be found analytically.

However, we first point out a possible problem. It is quite possible that in equation (15) two different values of θ will give the same value of λ. Hence when we invert (15) there may be more than one solution branch. The device will only be able to approximate one solution branch (it has no means to decide which branch it is on), and this may lead to an erroneous value of θ, and hence a miscalculation of $I_1 + I_2$. It is therefore important to make sure that the solution of (15) remains on one branch for all relevant angles.

3 Example

In this section we consider the configuration of probes shown in figure 3. This corresponds to the following choice of parameters:

$$\alpha_1 = 0, \qquad \alpha_2 = 0, \qquad \alpha_3 = 0, \qquad \alpha_4 = 0,$$

$$\beta_1 = 0, \qquad \beta_2 = 0, \qquad \beta_3 = 0, \qquad \beta_4 = 0,$$

$$r_4 = r_1, \qquad r_3 = r_2.$$

Without loss of generality we set $d = 1$. Equations (8) and (9) become

$$B_1 - B_4 - (2/r_1)I_3 = \frac{2r_1(I_1 + I_2)(r_1^2 - \cos 2\theta)}{1 + r_1^4 - 2r_1^2 \cos 2\theta}, \tag{17}$$

$$B_2 - B_3 + (2/r_2)I_3 = -\frac{2r_2(I_1 + I_2)(r_2^2 - \cos 2\theta)}{1 + r_2^4 - 2r_2^2 \cos 2\theta}. \tag{18}$$

Equation (10) becomes

$$\lambda = -\frac{r_1(r_1^2 - \cos 2\theta)(1 + r_2^4 - 2r_2^2 \cos 2\theta)}{r_2(r_2^2 - \cos 2\theta)(1 + r_1^4 - 2r_1^2 \cos 2\theta)} \tag{19}$$

This is a quadratic equation for $\cos 2\theta$, which is easily solved.

At this point (to avoid complicated formulas) we make our (arbitrary) choice of r_1 and r_2. We set $r_1 = 0.5$, $r_2 = 0.25$. Solving for $\cos 2\theta$ we find

$$\cos 2\theta = \frac{265 + 140\lambda - 3\sqrt{6889 + 6296\lambda + 1936\lambda^2}}{64(1 + 2\lambda)}. \tag{20}$$

Here we have chosen the solution branch through $\theta = 0$. Figure 4 shows λ as a function of $\cos 2\theta$. We see that the solution remains on this branch until $\cos 2\theta = 1/16$, or $\theta = 43.2^o = \theta_c$. Hence this bounds the maximum deflection for which we can accurately solve for the residual current in this case. Note also that, since $\lambda \to \infty$ as $\theta \to \theta_c$, if this angle is close to our desired range of operation we will have to approximate $g(\lambda)$ over a much greater range of values of λ. Thus it is important that θ_c is well outside the range of angles in which we are interested. We consider from this point deflections of up to 30^o. Figure 5 shows λ as a function of θ in this range. From (18) we now have

$$g(\lambda) = \frac{83 + 296\lambda + \sqrt{6889 + 6296\lambda + 1936\lambda^2}}{4(87 + 44\lambda - \sqrt{6889 + 6296\lambda + 1936\lambda^2})}, \tag{21}$$

and hence

$$I_1 + I_2 = (B_2 - B_3 + 8I_3)\left(\frac{83 + 296\lambda + \sqrt{6889 + 6296\lambda + 1936\lambda^2}}{4(87 + 44\lambda - \sqrt{6889 + 6296\lambda + 1936\lambda^2})}\right). \tag{22}$$

Figure 6 is a graph of g over the desired range. Since λ is measured we have now determined the current $I_1 + I_2$ exactly, providing we can evaluate (22).

Because of the appearance of square roots in (22) it may not be possible to evaluate it exactly with simple components. In this case we need to find the simplest approximation within the required accuracy, over the required range of angles. For example, suppose that we are interested in deflections up to 30^o. Then we need to approximate (22) over the interval $\lambda(0) \leq \lambda \leq \lambda(\cos 60^o)$, i.e. $-2.5 \leq \lambda \leq -1.3$ (see figure 5). One possible approximation would be a Taylor series about $\lambda = \lambda(0)$. However, this would give a poor approximation to (22) over the whole range of angles $0 \leq \theta \leq 30^o$. A Taylor series will give the best approximation near $\theta = 0$, but what is really required here is a uniform approximation to the desired accuracy over the whole range. Polynomial interpolation is a far better approach. In fig. 7 $g(\lambda)$ has been approximated linearly for angles up to 30^o with the polynomial $p_1 = 6.54601 + 1.92456\lambda$. The error in this approximation is shown in fig. 8. We see that the maximum error is about 7.5 percent over this range of angles. In fig. 9 $g(\lambda)$ has been approximated by the quadratic $p_2 = 10.7867 + 6.43345\lambda + 1.14893\lambda^2$ over the same range. The error, shown in fig. 10, can be seen to be less than 0.6 percent.

Other configurations can be treated in the same way. However, in most other cases it will not be possible to solve analytically for $\cos 2\theta$.

4 Conclusion

In principle it is possible to solve exactly for the residual current $I_1 + I_2$ and the deflection θ for any configuration of three probes or more. However, the problem in practice is to approximate this solution in a way that the detector can evaluate easily.

A significant simplification occurs if four probes are used in two pairs of equal and opposite probes. In particular, the deflection angle is now a function of only one variable, namely the ratio $\lambda = (B_1 - B_4 - (2\cos(\beta_1)/r_1)I_3)/(B_2 - B_3 + (2\cos(\beta_2)/r_2)I_3)$. This greatly simplifies approximations of this angle.

Having settled on such a configuration the procedure is as follows:

1 Make sure the choice of the remaining parameters is such that the solution of θ as a function of λ remains on one solution branch for all relevant angles.

2 Solve (either analytically or numerically) equation (10) for θ as a function of λ.

3 Substitute this solution into equation (9) to give $I_1 + I_2 = (B_2 - B_3 + (2\cos(\beta_2)/r_2)I_3)g(\lambda)$.

4 Approximate $g(\lambda)$ to the desired accuracy over the desired range of values of λ.

The optimal configuration now depends on the types of approximation possible.

In the previous section we examined a configuration in which steps 1-3 above can be performed easily and exactly. We then have an exact expression for the residual current valid for all angles θ. What remains is to approximate this exact solution with functions which the detector is able to evaluate (eg polynomials). With this configuration a linear approximation gave a maximum error of about 7.5 percent for angles of deflection up to 30°, and a quadratic approximation gave a maximum error of 0.6 percent over the same range.

If the device is such that it can perform division and multiplication of the probe readings $B_1, \cdots, B_4$, I_3 then ANY probe configuration can achieve ANY desired accuracy simply by using interpolating polynomials of higher degree (subject to the constraint that the solution of equation (15) remains on one branch). In this case other factors will no doubt influence the probe design.

If the device is such that it can only add, subtract and scale the probe readings, then a linear approximation to $g(\lambda)$ must be used. In such a situation various configurations could be tried to see if the error in a linear approximation can be made small enough for the desired range of deflections.

Finally we note since our primary concern was the residual current, we restricted ourselves to approximating the sum $I_1 + I_2$. In fact a similar procedure works to give an approximation of the difference $I_1 - I_2$. By using both these approximations the individual currents I_1 and I_2 could be calculated if necessary.

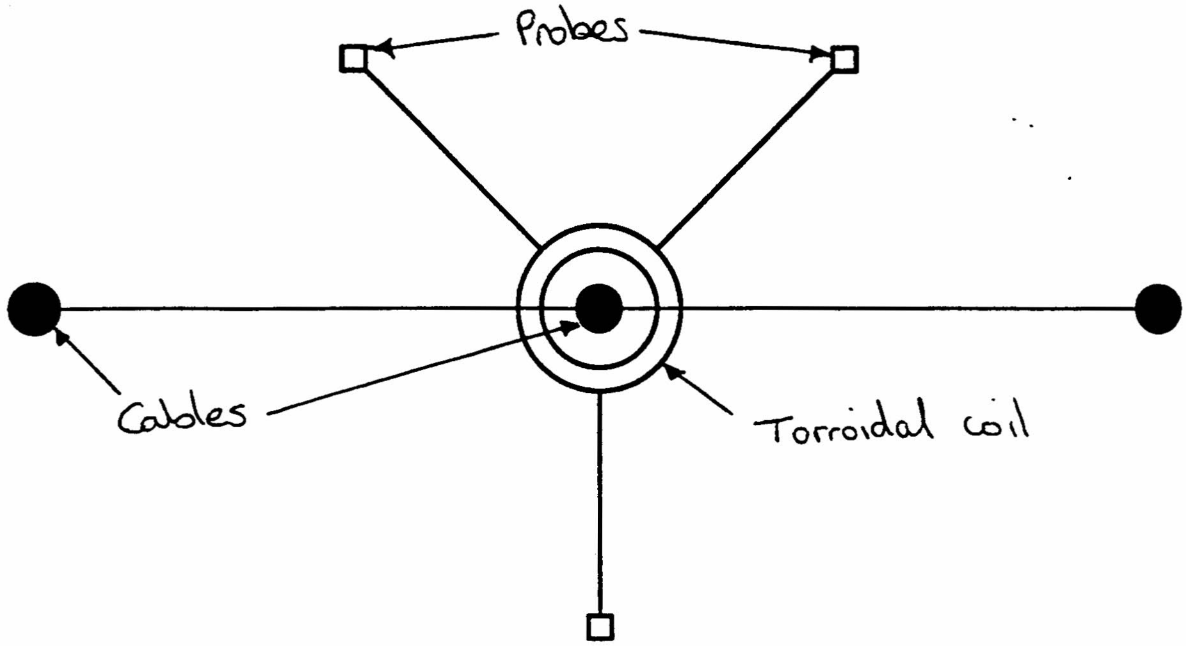
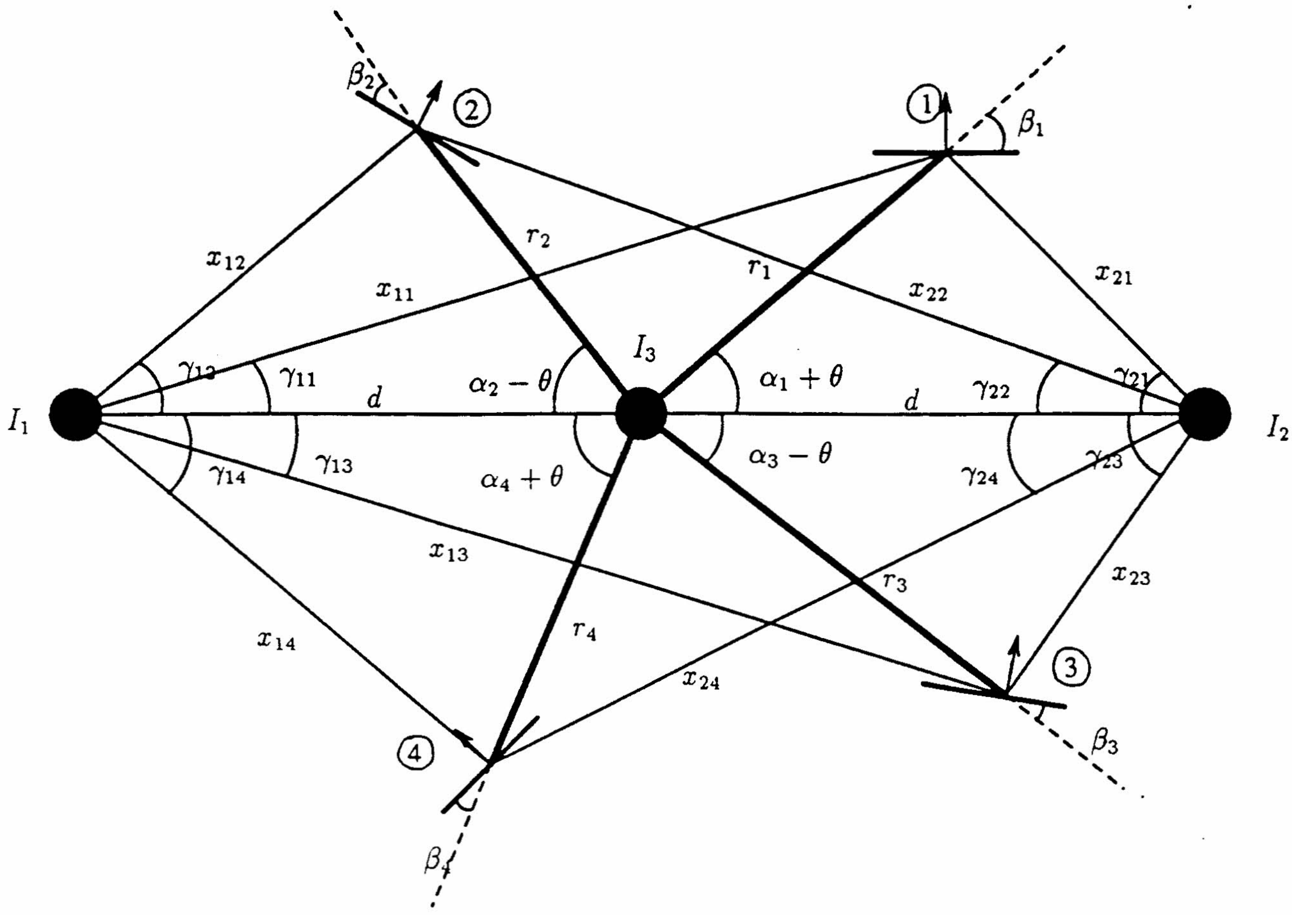


Figure 1



I_1
I_2
I_3
x_{12}
x_{11}
x_{21}
x_{22}
x_{13}
x_{14}
x_{23}
x_{24}
r_1
r_2
r_3
r_4
d
d
γ_{12}
γ_{11}
γ_{13}
γ_{14}
γ_{21}
γ_{22}
γ_{23}
γ_{24}
$\alpha_2 - \theta$
$\alpha_1 + \theta$
$\alpha_4 + \theta$
$\alpha_3 - \theta$
β_1
β_2
β_3
β_4
1
2
3
4

Figure 2.

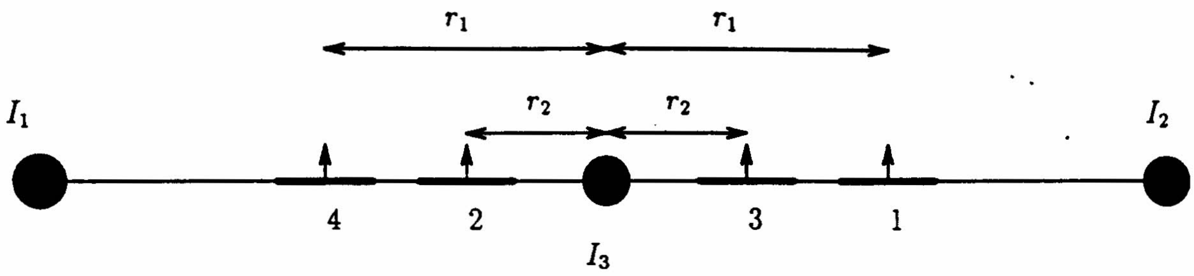


Figure 3.

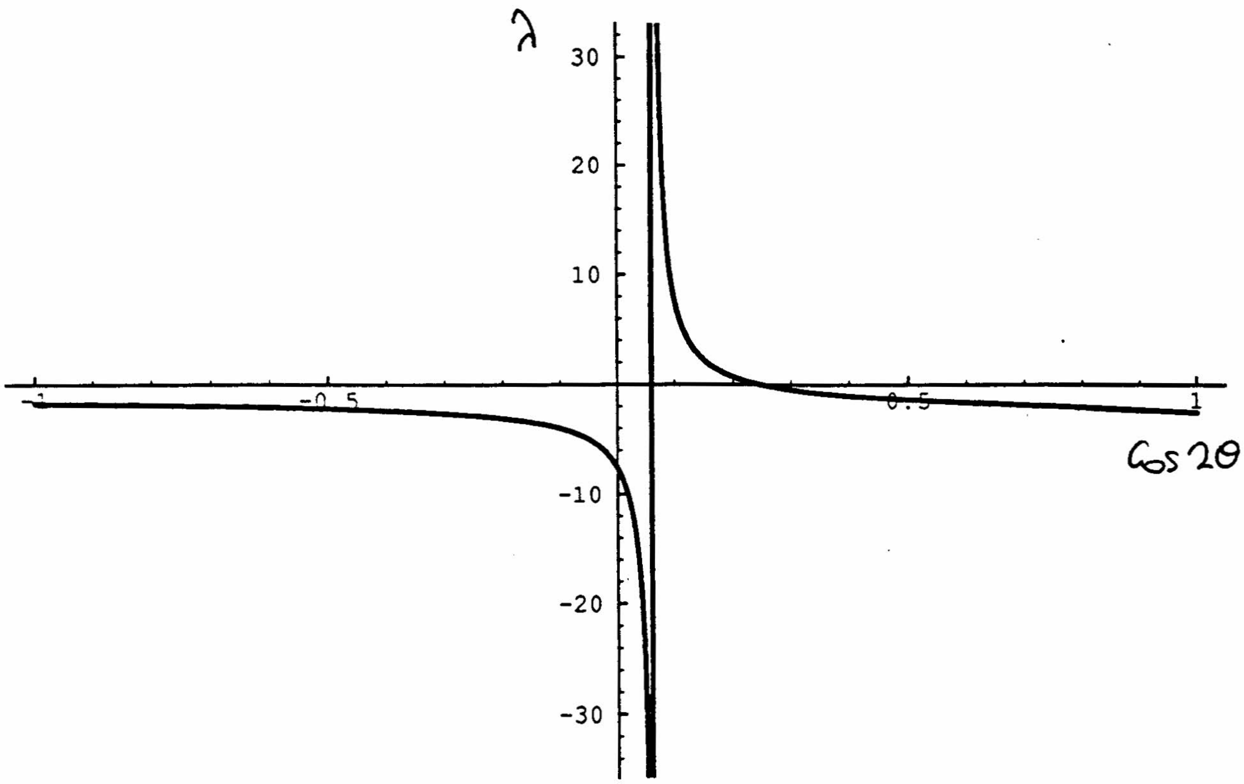


Figure 4

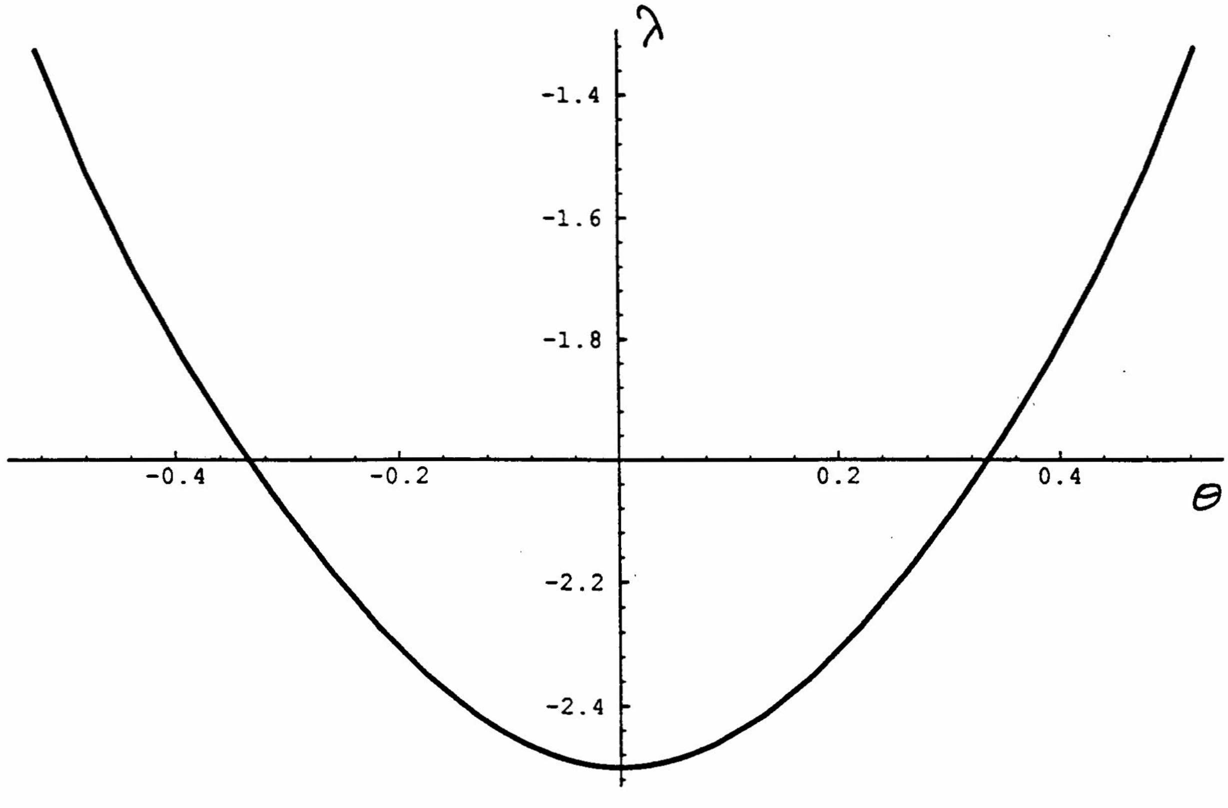


Figure 5

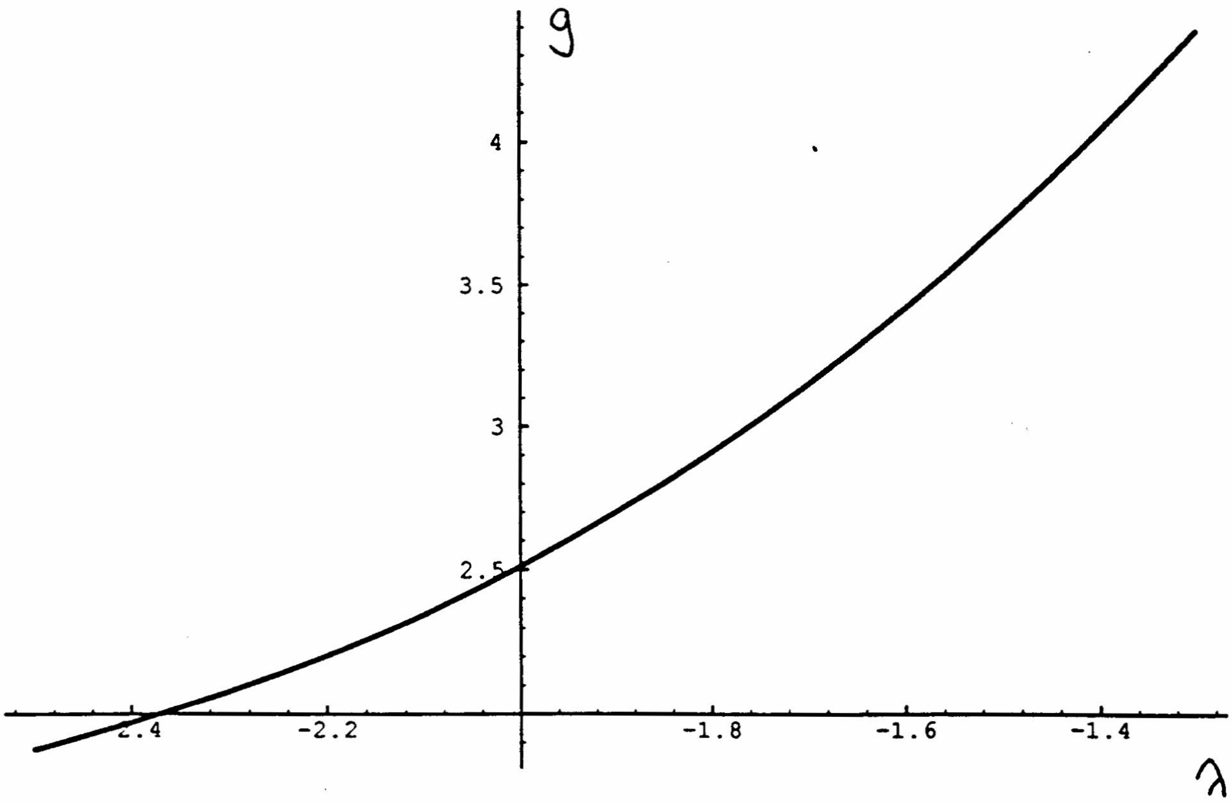


Figure 6

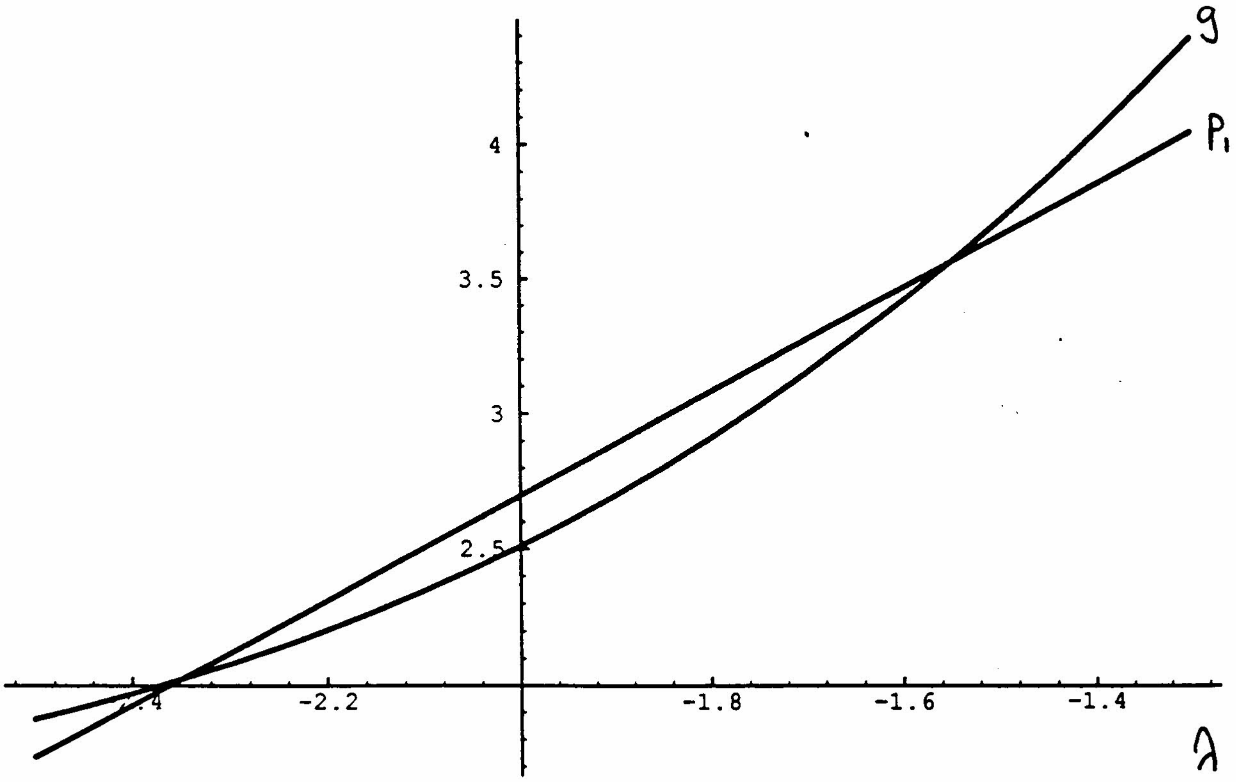


Figure 7

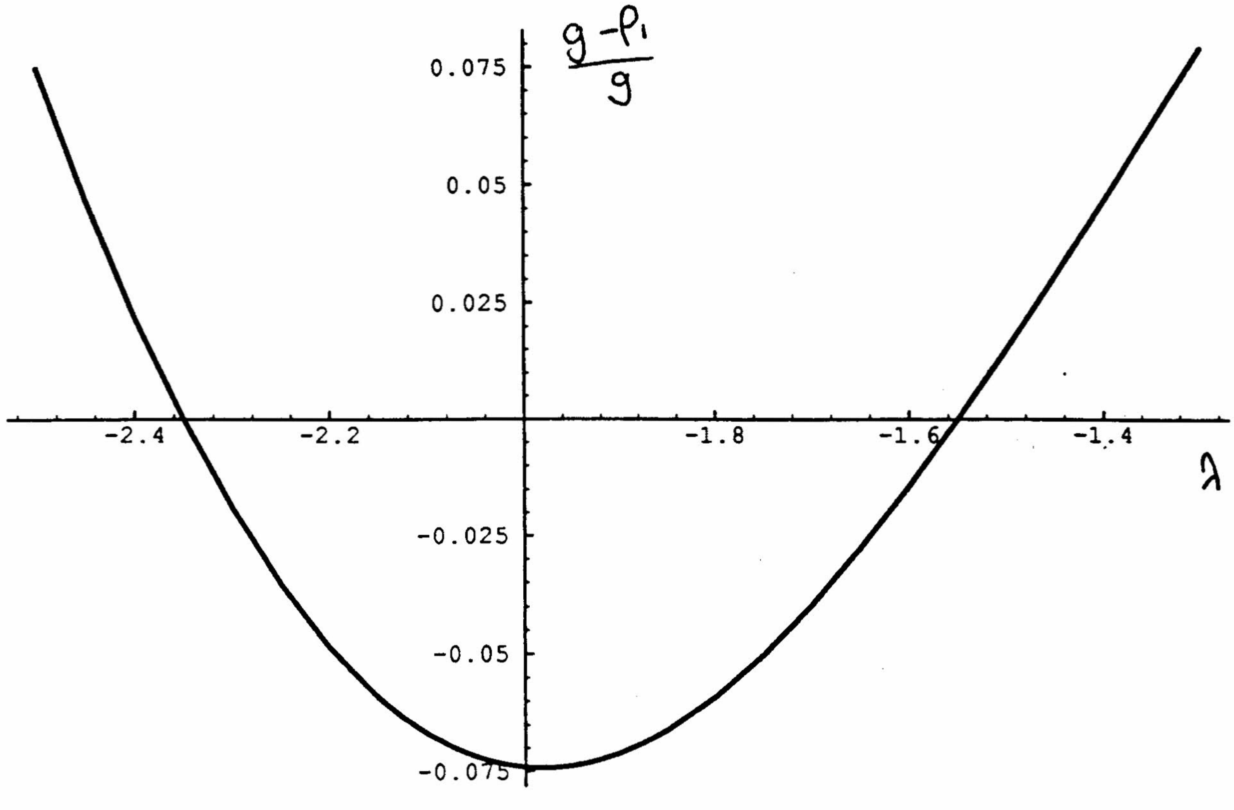


Figure 8

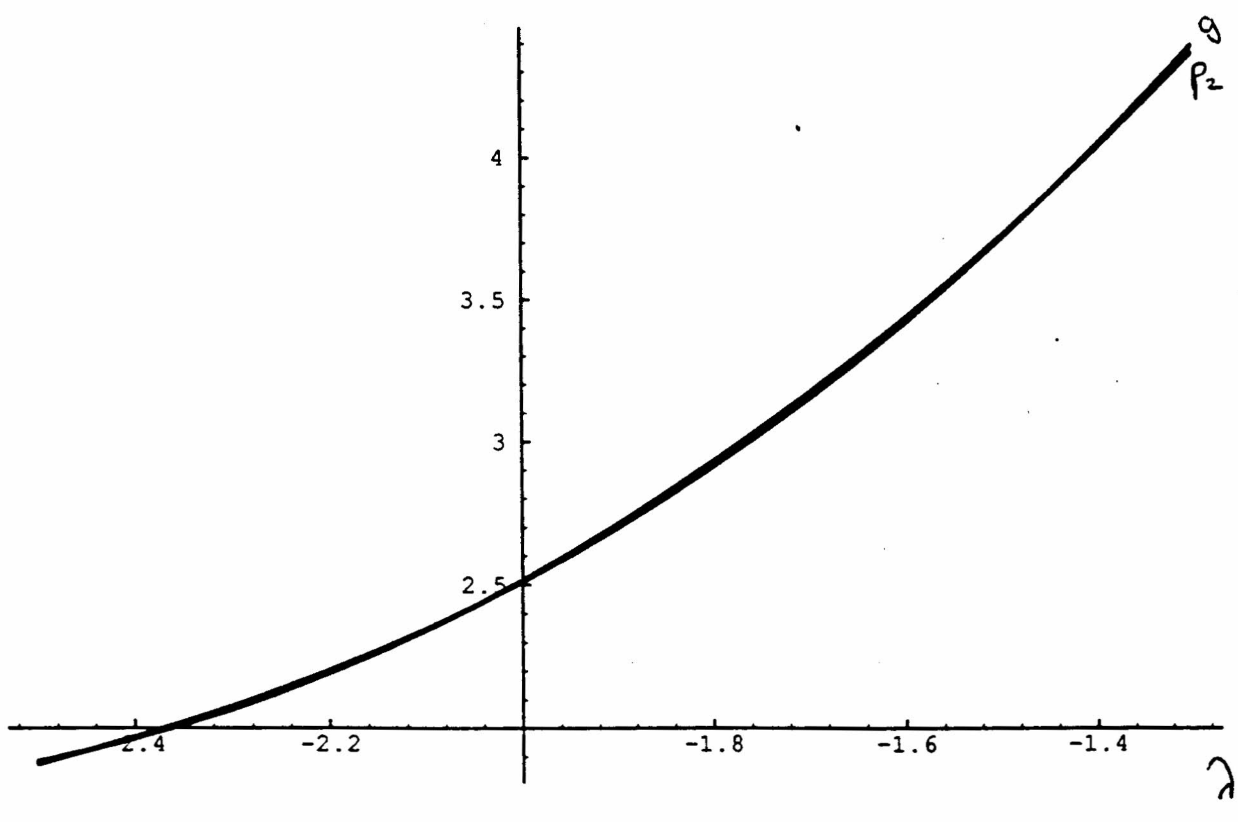
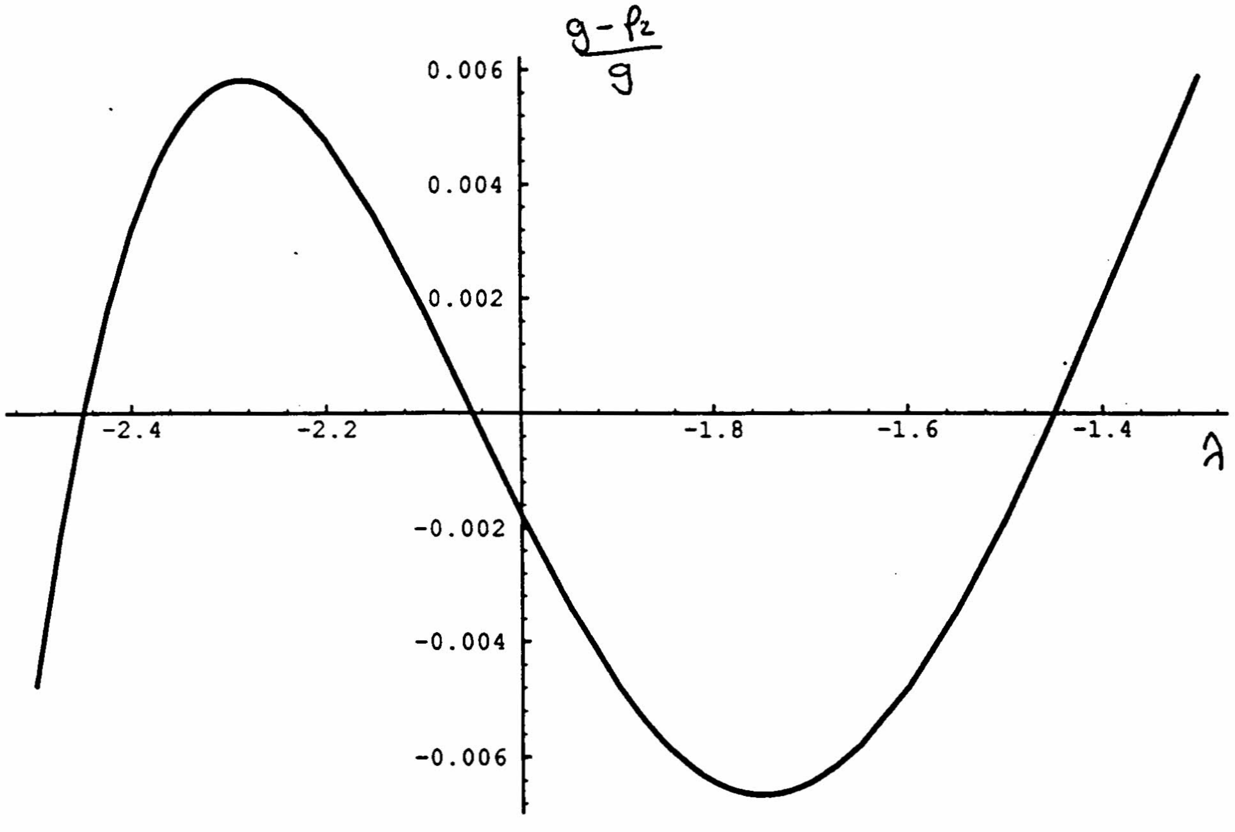


Figure 9



$\frac{g - \rho_2}{g}$
0.006
0.004
0.002
-0.002
-0.004
-0.006
-2.4
-2.2
-1.8
-1.6
-1.4
λ

Figure 10